Contemporary Mathematics in Context: A Unified Approach (Core-Plus Mathematics Project- CPMP)

Levels: 9-12

Developers: Arthur Coxford, James Fey, Christian Hirsch, Harold Schoen, Gail Burrill, Eric Hart, Brian Keller, Ann Watkins, Mary Jo Messenger, and Beth Ritsema

Publisher: Glencoe/McGraw-Hill

Review Materials: Courses 1, 2, and 3 were reviewed in published form. Course 4 is now in published form, but was not reviewed. The Teacher's Guide (which includes student facing pages), Teaching Resources, and Assessment Resources for Courses 1-3 were reviewed. Also available are: Implementing the **Core-Plus Mathematics** Curriculum, Scope and Sequence for Courses 1-4, Calculator Software and Guide, Reference and Practice Books for Courses 1-3, and Assessment and Maintenance Worksheet Builder CDs.

Format/Description: This is a complete four-year secondary school mathematics curriculum for all students consisting of a three-year core program plus a flexible fourth-year course continuing the preparation of students for college mathematics. The curriculum is centered around four strands: algebra/functions, geometry/trigonometry, statistics/probability, and discrete mathematics. Mathematical modeling is a central focus throughout the curriculum. Each of Courses 1-3 contains six *units* which focus on different mathematical topics together with a seventh Capstone unit. Course 4 consists of ten units that formalize and extend the core program with a focus on the mathematics for students who plan to continue their study of mathematics in college. The units integrate mathematical topics from different mathematical disciplines (see **Content Overview** below). In addition, the text is written in a way that often connects the current mathematics and contextual settings with mathematics and contents seen previously in the curriculum. Units take from four to six weeks to complete. Each unit consists of three to six lessons. Each lesson is organized around several multi-day investigations. Each lesson consist of five parts: the *Launch*- presentation of a problem situation with several general questions for the entire class to think about; the *Exploration*- students investigate focused problems in small collaborative groups; the *Checkpoint*- groups share and summarize findings with the full class; and *On Your Own*- students work individually on problems. The fifth part is the *MORE* (Modeling, Organizing, Reflecting, Extending) out-of-class activities intended to be done individually. In this section in particular, tasks are differentiated in difficulty in order to accommodate variation in student ability and interest. The Capstone is a thematic two-week project-oriented unit that explores a context involving reference to many of the mathematical topics in that year's curriculum.

The Teacher's Guide includes instructional notes for each page of student materials and suggestions for implementation of the materials in the classroom including specifics on the role of the teacher, unit overviews, learning objectives, suggested timelines, lists of materials needed, suggested assignments, solutions to student activities and problems. Teacher Resources include blackline masters and transparencies. Quizzes, end-of-unit assessments, take-home assessments, projects, and mid-term and final exam tasks are included in the Assessment Resources. For additional information concerning the format and other valuable information, see: Implementing the **Core-Plus Mathematics** Curriculum, and Contemporary Mathematics in Context Sampler.

Pedagogy: The curriculum delivery focus is on student learning. The student materials are written in a way that promotes active student participation. Students spend most of their time in class working collaboratively in small groups or individually with the teacher as facilitator or "coach." However, as indicated above, some time is devoted to full class discussion with teacher as director (Launch) or moderator (Share and Summary). Students are *consistently* asked to explain their reasoning in each lesson. The core mathematical topics are accessible to all students. Differences in interest and performance are accommodated by the depth and level of abstraction to which the topics could be pursued and by the choices of homework tasks and projects. The booklet Implementing the Core-Plus Mathematics Curriculum contains descriptions and suggestions with regard to the collaborative learning and classroom discourse. Included in the Teachers' Guide is information on and suggestions for the personal Math Toolkits that students construct as they progress through each course. These toolkits provide students with a summary of the important mathematical concepts and methods that they have learned. Each student retains his/her toolkit as a reference in the next course.

Technology: Graphing calculators are an essential component of this curriculum and are used heavily in most units. In particular, the TI-82, TI-83, and TI-92 series are supported in the student and teacher materials. Downloadable programs for the TI-82-83-92 are available, as is the booklet, Calculator Software Guide. Students use the table, graph, programming, list, statistics, plot, (recursive) sequence, and matrix capabilities of these calculators. CPMP has developed spreadsheet software for the TI-92. Calculator or computer spreadsheet software is necessary for Unit 10 of Course 4.

Assessment: The assessment program is explained in the booklet Implementing the Core-Plus Mathematics Curriculum. The program includes curriculum-embedded informal assessment by the teacher when students are working collaboratively or individually. There are several points in each investigation where the teacher can more formally assess students either orally or in writing. In particular, the *Checkpoints*, *On Your Own*, and *MORE* tasks provide such opportunities. Student journals and portfolios are also suggested. Supplementary materials incorporating end-of-lesson quizzes, end-of-unit exams (in-class and take-home), and projects are contained in the Assessment Resources. There are Forms A and B for all quizzes and tests. Scoring rubrics and suggestions for grading are also included. Assessments are available on CD for Courses 1-3 in order to allow teachers to customize assessments for their classes.

Content Overview: The first four NCTM Standards: Mathematics as Problem Solving, Mathematics as Communication, Mathematics as Reasoning, and Mathematical Connections are addressed heavily in every unit. Students are consistently asked thought-provoking questions and required to explain their reasoning. Many of the questions are open-ended. The mathematical topics are strongly integrated. Connections among the units and the strands mentioned above are made by repeating contexts, such as bungee jumping or population growth; revisiting mathematical topics from a different point of view, such as symmetry or matrices; by habits of mind, such as visual or recursive thinking; and by returning to the consistent themes of data, multi-representation, shape, and change. The development of mathematical content is done largely through mathematical modeling of real-world contexts and reflection on underlying mathematical structures. The contexts have been carefully chosen to appeal to students without diminishing the importance and usefulness of the mathematics. Students become acquainted with mathematical definitions and terminology in the course of solving problems.

As stated above, the curriculum is centered around four "strands." As the developers express it, **algebra and functions** "is designed to develop student ability to recognize, represent, and solve problems involving relations among quantitative variables." **Geometry and trigonometry** is aimed at developing "visual thinking and students' ability to construct, reason with, interpret, and apply mathematical models of patterns in visual and physical contexts." Statistics and **probability** develops "student ability to analyze data intelligently, to recognize and measure, variation, and to understand the patterns that underlie probability situations**."** **Discrete mathematics** "develops student ability to model and solve problems involving sequential change, enumeration, decision making in finite settings, and relationships among a finite number of elements." These descriptions are taken from the booklet <u>Implementing the Core-Plus Mathematics Curriculum</u>. This booklet includes other details concerning these strands. Another source of information regarding content in the curriculum is the <u>Scope and Sequence</u>. Our description of what students completing this curriculum will know and be able to do is given in terms of these strands. Some topics, for example, NOW-NEXT relations, could be listed in more that one strand; in **algebra and functions** and also in **discrete mathematics**, for instance. The decision has been made to list most topics only once within each course. It should be emphasized that these strands are interwoven throughout units and throughout the courses. Moreover, the present description does not necessarily describe topics in the order in which they are encountered in the curriculum. For the most part, topics which are briefly mentioned but not emphasized to any degree (e.g. "indegree" of a vertex in a digraph) are omitted. In some places, we simply state that students are "exposed" to such topics (e.g. the concept of a permutation in Course 1). In contrast, in what follows, **when it is said that a student will be able to do a task or has a strategy to solve a problem, it is implied that the student will also be able to explain what they are doing and why they are doing it.** One additional facet of the curriculum appertains to the use of technology; in particular, graphing calculators. In each course, emphasis is placed on students' ability to recognize and employ the capabilities of the graphing calculator in relation to the mathematical topics and contexts presented. The technology is utilized in ways which facilitate the student's insight into the mathematical concepts and which provide additional strategies for solving problems. The use of technology does not replace understanding with dependence upon senselessly applying a calculator algorithm.

<u>Course 1:</u>

<u>**Algebra and functions**</u>**:** Students will be able to identify key variables in a variety of situations. Students will be able to identify patterns of change and model them using NOW-NEXT recurrence relations, tables, graphs, verbal descriptions and symbolic function rules relating the variables. Both linear and nonlinear relationships are encountered. However, linear and exponential relationships are featured in Course 1. The linear situations students will be able to model include many real-world contexts as well as using lines to describe the shape of data on a scatterplot. In addition to using the linear regression capability of the calculators to fit lines to

linear data patterns, students will be able to write an equation of a line given the line's slope and y-intercept or given two points on the line. They will be able to interpret the meaning of the coefficients of a linear equation in real world terms given the context of the problem. For instance, they will be able to relate the concept of slope to the geometric inclination of the graph of a straight line and they will be able to interpret the same coefficient as the rate of change of one variable with respect to the other in a different context. They will be able to recognize a linear relationship represented by a table, by a graph, and by symbolic forms, particularly $y = ax + b$, and construct one representation from another. They will be able to solve linear equations in one variable using standard balancing or "undoing" algorithms. They will be able to use the commutative property of addition and the distributive property of multiplication over addition to rewrite linear equations in one variable in mathematically equivalent forms. They will be able to write linear inequalities in one variable from contexts and be able to solve linear inequalities in one variable by inspection, by using tables, and by using some graphical methods. They will also be able to determine equivalence of linear equations with two variables using tables and graphs. Students will be able to recognize exponential growth (or decay) by examining tables and graphs of situations where something is growing (shrinking) at an ever-increasing (decreasing) rate. They will understand the equation $y=a(b^x)$, where $1< b$ and also where $0<b<1$. They will use graphs and tables to solve exponential equations. They will be able to use exponential properties to model compound growth (e.g. compound interest). They will be able to model many real-world situations using exponential growth and decay models, and be able to use technology to fit an exponential curve to data on a scatterplot. They will be able to compare exponential models for different positive values of a and b and make comparisons between linear and exponential models.

Geometry and trigonometry: Students will enhance their planar and spacial visualization skills. They will be able to define, recognize, and classify shapes such as squares, rectangles, parallelograms, rhombi, trapezoids, kites, regular polygons, prisms, pyramids, (and possibly tetrahedra, octahedrons, dodecahedrons, and icosahedrons), cones, and cylinders. They will understand and be able to use mathematical terms such as face, edge, base, solid, shell, skeleton, and rigid (and possibly oblique and adjacent face) to describe three-dimensional objects. They will be able to create face-views and isometric and perspective drawings of objects made of cubes. They will understand how nets may be used to describe three-dimensional objects. They will be able to recognize reflection symmetry in both two and three dimensions. Students will be able to describe the symmetry of plane tessellations, polygons, and frieze (strip) patterns using the language of isometric transformations (translations, rotations, reflections, and glide reflections). They will be able to use polygons and three-dimensional shapes to model some real-world situations and reason about some spatial situations. They will enhance their skill of measuring and describing the size of an object through the use of length, perimeter, diagonal measure (of a rectangle), area, volume, and surface area (of planar, locally flat three-dimensional objects, and the cylinder.) They will understand area as the number of square units necessary to fill a plane region. They will understand volume as the number of cubes needed to fill a three-dimensional region. They will know and be able to use several standard formulas for area (e.g. triangle, parallelogram, circle) and volume (e.g. cylinder, prism). They will know (have seen at least suggestions toward a proof of) and be able to use the Pythagorean Theorem. In Course 1 students develop their reasoning abilities by using inductive reasoning, providing counterexamples, and explaining or justifying their responses throughout all strands of mathematics (formal proof construction is begun in Course 3).

Statistics and probability: Students will enhance their skills in collecting, organizing, displaying, and analyzing data. They will understand, be able to find, be able to use, and be able to compare the three measures of central tendency: the mean, median and mode. They will be able to display and interpret distributions of data using histograms, plots over time, bar graphs, standard and back-to-back stem and leaf plots, box plots, frequency tables, and scatterplots. They will be able to choose appropriate representations of data and compare representations. They will understand and be able to compute several measures of variation including the mean absolute deviation, percentiles, and the range and interquartile range. They will be able to use the five number summary (minimum, lower quartile, median, upper quartile, and maximum) to describe a distribution. They will be able to recognize reflective symmetry and outliers in a distribution and be able to explain the significance of each. They will know the effect of a linear transformation (not given in those terms) of data on the mean, median, and mean absolute deviation of a data set. They will be able to use technology (involving random number generators), random digit tables, and experimental methods to simulate "real world" and other probabilistic situations. For example, they will be able to simulate the distribution and expected value of a geometric random variable, and, hence establish empirical probabilities. (The term geometric random variable is not used.) They will understand what a trial of a simulation is. They will understand what equally-likely probabilities are and establish some simple theoretical probabilities, such as getting a head when flipping a fair coin. They will informally understand expected value in many instances. They will be able to estimate expected value by computing averages in simulations. They will have explored properties of random digits and should informally be able to explain what "random" means. They will understand the Law of Large Numbers as meaning, for example, that as the number of trials increases in a binomial random variable experiment, the actual percentage of successes approaches the theoretically expected percentage of successes. (The term binomial random variable is not used.) They will be exposed to the concept of a permutation.

Discrete mathematics: Students will be able to create and utilize vertex-edge graphs to model many real world situations including finding efficient routes (using Euler paths), managing conflicts (by coloring vertices), and scheduling (using digraphs and PERT charts). Students are exposed to graphs that contain loops, but most graphs do not contain loops. In this mathematical context, students will have experienced the three-part mathematical question: is there an answer (existence of a solution); if there is an answer, how do you determine an answer (algorithm formulation); and if there is more than one answer, how do you find the best answer or most efficient solution method (optimization)? They will have investigated uses and strategies for coloring the vertices of a graph. They will have investigated criteria for the existence of Euler circuits and paths and systematic procedures for vertex coloring. They should have discovered and be able to use Euler's theorem. They will be able to represent and analyze graphs using adjacency matrices. They will be able to use at least one strategy (algorithm) for constructing an Euler path and an Euler circuit and they will be able to Eulerize a connected graph. They will have at least one strategy for finding critical paths in weighted digraphs. They will be able to use at least one strategy for scheduling a project (**PERT**) using the notions of prerequisite tasks, the EST (earliest start time), EFT (earliest finish time), LST (latest start time), and slack time. They will have investigated scheduling projects with uncertain task times and been exposed to at least one algorithm for scheduling such projects. They will understand the degree of a vertex.

Course 2:

Algebra and functions: Students will be able to write systems of linear equations of the form ax + by = c that model real-life linear relationships between two variables. They will be able to solve these systems using graphing and linear combinations of equations (which corresponds to the method of elimination or addition/subtraction method, but is developed by examining the graph of the sum of non-zero multiples of linear equations). Students will be able to graph equations of the form ax + by = c by making a table of points and by solving for y (when lines are not vertical). They will be able to describe patterns of change in power models of direct variation (in particular, $y = ax^2$ and $y = ax^3$) and inverse variation ($y = a/x$ and $y = a/x^2$) by looking at tables and graphs of those relations and making connections with the symbolic forms. They will be able to compare these patterns with those in linear and exponential models, as well as other power models. Students will investigate the properties of quadratic models ($y = ax^2$, $y = ax^2+c$, $y = ax^2 + bx + c$, introduced as the sum of power and linear rules). They will be able to predict the shape of a graph from its symbolic rule. They will be able to write algebraic rules to model situations involving time/height (e.g. the flight of a springboard diver) and income/profit in order to answer questions related to the context. Students will compare various quadratic rules and corresponding tables and graphs. Using a table or graph, they will be able to estimate the root(s) of a quadratic equation. They will be able to determine the number of roots by looking at the graph of a quadratic model. They will be able to solve equations of the form $ax^2+b=c$, $\sqrt{x} = k$, $ax^3 + b = c$, and $\sqrt[3]{x} = k$ using a table of values, a graph, or by reasoning with the symbolic form (e.g. $4000 = 20h^2 + 300 \Rightarrow 3700 = 20h^2 \Rightarrow 185 = h^2 \Rightarrow \pm\sqrt{185} = h$). Students will be able to use radicals and their properties. For example, they will know that

$$\sqrt{ab} = \sqrt{a}\sqrt{b}, \text{ when } a, b > 0.$$

Similarly, they will know that the expression

$$\sqrt{a^2 + b^2} = a + b$$

is generally false. They will know and be able to use some fractional powers (such as $^1/_2$ and $^1/_3$) in power models. They will know that $x^{-n} = 1/x^n$, where n is a positive integer. They will know and be able to use the laws of exponents for integers (and at least the fractional powers they have studied). They will be able to describe the table and graph patterns of $y = \sqrt{x}$ and $y = \sqrt[3]{x}$.

Geometry and trigonometry: Using coordinate geometry and programming techniques, students will investigate some of the mathematics related to geometric shapes which is involved in computer graphics. They will develop formulas for the distance between two points, midpoint coordinates, and slope of a line, and then program their calculators to perform these computations. Using these programs and additional software, they will analyze properties of geometric shapes to make conjectures and then determine whether they are rectangles, squares, isosceles triangles, or parallelograms. They will be able to represent transformations (translations, line reflections, rotations about the origin, glide reflections, and size transformations centered at the origin) using coordinates, matrices, and geometric descriptions.

They will examine the effects of these transformations and combinations of transformations on distance, angle measure, parallelism, and area. Students will explore the fundamental theorem of proportionality (not stated as such) as they investigate relationships between length, perimeter, surface area, and volume of similar figures. They will recognize that geometric form can be essential to the functioning of an object (e.g. triangles are rigid, quadrilaterals can pivot at vertices, circles turn easily at their centers). Using a pantograph, they will be able to create similar figures with scale factors. They will explore patterns in measures of angles and altitudes of triangles with a variable-length side.

Students will be able to find the sine, cosine, and tangent of an angle in a right triangle. They will be able to articulate why these ratios are independent of the lengths of the sides of a right triangle with a given acute angle. Using trigonometric ratios, they will be able to calculate the length of a side and the measure of an angle (using the inverse trigonometric function keys on the calculator) of a right triangle, and solve indirect measurement problems. Beginning with an analysis of situations involving pulleys and sprockets (rotating circular objects) to determine linear and angular velocity, students move into an investigation of the graphs of the trigonometric functions and models of periodic motion. They will be able to connect the trigonometric ratios of sine and cosine with circular motion as they build a model of a Ferris wheel and explore the periodic nature of its movement. They will be able to sketch graphs of the sine and cosine functions that model periodic phenomena. They will be able to graph y=AsinBx and y=AcosBx using the graphics calculator, and examine the effects of the parameters on the amplitude and period of a given function. They will be able to compare and contrast the graphs of the sine and cosine functions. Students will use both radian and degree measure with trigonometric functions.

Statistics and probability: Students will further explore the association between pairs of variables through interpretation of scatterplots, scatterplot matrices, and correlation matrices. They will investigate numerical measures to describe the strength of linear association. In particular, they will be able to compute and interpret the correlation coefficient for ranked and unranked data. They will recognize the importance of combining graphical and numerical analysis, since outliers (influential points) can affect results. They will examine the possible cause-and-effect relationship between two variables, and be able to give possible reasons for correlation. They will be able to use a regression line for a set of data to make predictions and compute the error in prediction. They will become familiar with the method of least squares, used by the calculator to determine the regression line. Students will explore waiting-time distributions (also known as geometric distributions) through simulation and experimentation, as well as theoretically. They will be able to construct frequency distributions and histograms for waiting-time distributions, and will discover they have the same basic shape. They will be able to estimate average waiting time using histograms and frequency tables. They will be able to distinguish between independent trials and those that are not independent. They will be able to find the probability of two independent events both occurring, using an area model and the Multiplication Rule (p(AandB)=p(A)· p(B)). They will be able to determine whether or not two events are independent, using p(A)=p(A|B). They will be able to explain how the word "expect" is used in terms of probability. Students will be able to construct and analyze probability distribution tables and graphs. They will be able to use probability notation. They will be able to determine a rare event (i.e. 0<p(event)#0.05). They will be able to compute the expected

value for games of chance and insurance (called fair price) and for probability distributions. They will be able to compute the expected value for a waiting-time distribution (E.V.=1/p).

<u>Discrete mathematics</u>: Students will be able to create a matrix model for organizing and displaying data in many real world situations (business, archeology, sociology, sports, ecosystems). They will understand the advantages and disadvantages of displaying data in matrices. They will be able to analyze matrices to draw conclusions about data. They will be able to perform matrix operations using addition and subtraction of matrices, row sums, and scalar multiplication, and interpret results. They will be able to use matrix multiplication, including positive powers of matrices, to help make decisions in a variety of contexts. They will study algebraic properties of matrix operations. They will be able to represent systems of linear equations with matrices. They will be able to write a matrix equation of the form AX=C and solve by multiplying by the inverse matrix A^{-1} (using the x^{-1} key on the calculator). They will be able to use this method to solve systems of linear equations (when the inverse of the coefficient matrix exists). They will be able to use matrix representations for line reflections, translations, rotations and "size" transformations (dilations) to write calculator programs that simulate computer animations. Students will extend their knowledge of vertex-edge graphs to represent and analyze a variety of real-world situations. Given a network of vertices and weighted edges, they will be able to apply and analyze algorithms that find a minimal spanning tree. They will be able to find a shortest path from one vertex to another, or a shortest Hamiltonian circuit (Traveling Salesperson Problem). They will understand why even with a computer a brute-force method is limited. Given a table or matrix, they will be able to create a weighted graph and given a weighted graph they will be able to create a distance matrix.

<u>Course 3:</u>

<u>Algebra and functions</u>: Students will be able to work with relations (both equalities and inequalities) involving several linked variables; sometimes together with additional parameters. Many relations involve three variables, but several involve more than three. In building or analyzing models, they will describe linkages using symbols, tables, and/or graphs. They will be able to identify how changes in one variable affect the value of another variable. They will be able to construct linear relationships such as z = ax + by from many contexts. They will be able to solve equations using the "properties of equality"(a+b = c implies a = b-c and ab=c implies a=b/c, provided c≠0). They will have at least one strategy for solving quadratic inequalities such as $2x+3>x^2$ (e.g. by graphing $y=2x+3$ and $y=x^2$ on the graphing calculator and determining when the first function is higher). They will be able to employ a strategy for solving linear inequalities in one variable using the "properties of inequality" (a+b<c implies that a<c-b and ab<c implies a<c/b if b>0.) They will understand what a system of equations is and at least one method to solve a two-dimensional system. In particular, they will understand a standard linear programming model with an objective function, constraints, and feasible points. They will know that the set of feasible points for a linear programming problem in two variables is a plane region. In particular, they will know that the set of points that satisfies a linear inequality in two variables is a half-plane, which can be found by graphing the corresponding linear equality and experimentally determining the half-plane. In addition, students will know that if the objective function can be optimized, an optimum occurs at a vertex of the feasible region.

Students will use the field properties for real numbers to justify the equivalence of algebraic equations. They will use algebraic reasoning to derive the quadratic formula. Students will also develop greater facility with algebraic operations with polynomials, including adding, subtracting, multiplying, factoring, and solving equations.

Students will be able to identify functions as variations of basic families of functions. Students will understand a function as a relation where there is exactly one output value (y-value) for each given input value (x-value). They will understand the domain and range (output values) of a function. They will be able to determine both practical (contextual) and theoretical (implicit) domains and ranges of many functions. They will be able to use function notation, such as f(x) = an expression, and determine when a graph or a table expressing a relation between two variables represents a function. They will be able to recognize and use functional relationships in many contexts. They will be able to categorize many functions. Students will understand that the graph of the function y=-f(x) is a reflection across the x-axis of the graph of f(x), that the graph of y=f(x)+c is a vertical translation of the graph of y=f(x), that the graph of y=cf(x) is a vertical stretch of the graph of y=f(x), when c>0, and that the graph of y = f(x - c) is a horizontal translation of the graph of f(x). They understand the absolute value function and that the sine and cosine functions are useful in describing periodic motion. They will be able to describe the effect of the parameter c in y=sin(cx) and y=cos(cx). They will know what polynomials are and what the degree of a polynomial is. Students will understand the basic operations of adding, subtracting, multiplying and dividing functions. They will be able to write equivalent symbolic expressions for functions and understand the value of "unsimplified" expressions as descriptive of a particular context. They will deepen their skill with the basic properties of addition (subtraction), and multiplication (division) in terms of associativity, commutativity, and the distribution of multiplication over addition in the context of operations with functions. They will be able to factor simple quadratic function expressions without using the quadratic formula. Students should be able to prove and use the quadratic formula to find the zeros of a quadratic, understand the relationship between the zeros of a quadratic and its factors, and understand that the expression

$$\frac{\sqrt{b^2 - 4ac}}{2a}$$

is the distance from the zeros of a quadratic to the axis of symmetry, provided a>0 and

$$\frac{\sqrt{b^2 - 4ac}}{2a}$$

is real. They will have at least two strategies for finding the zeros of a quadratic function using the graphing calculator. Students should realize the importance of x-intercepts, lines of symmetry, and maximum or minimum values in describing or determining the graph of a quadratic function. They should be able to determine the equation of the line of symmetry of a quadratic function.

<u>Geometry and trigonometry</u>: Students will enhance their skills using both deductive and inductive reasoning. They will understand the difference between inductive and deductive reasoning as well as the roles of conjecture, assumptions, mathematical proof, conclusions, and counterexamples. They will be able to make conjectures and apply simple deductive reasoning to conditional statements concerning angles formed by intersecting lines and angles formed by transversals of parallel lines. They will know the terms conditional and implication and how to use reasoning chains involving *modus ponens*. They will know, be able to reason with, and be able to use theorems concerning parallel lines and angle sums of polygons. They will know, be able to reason with, and be able to use the concepts of vertical and supplementary angles. Students will know and be able to use the four standard theorems to prove triangles congruent (SSS, SAS, ASA, and AAS). They will know and be able to use the three standard similarity theorems to prove triangles are similar (SSS, SAS, and AA). These theorems are derived as consequences of the Law of Cosine and Law of Sines which are also established. They will know and be able to use necessary and sufficient conditions to show a quadrilateral is a parallelogram. They will know and be able to use properties of special quadrilaterals such as rhombi and kites. They will know and be able to use the midpoint connector theorem. They will be able to provide a valid deductive argument using previously proved theorems for a given conjecture.

Students will be able to find the period and amplitude of sine and cosine functions given a graph or an equation, and move between equations and graphs of sine and cosine functions. They will also be able to write equations of trigonometric functions that model problem situations. Students will enhance their understanding of the Pythagorean theorem. For example, they will be able to prove cases of the Pythagorean identity $\sin^2(x)+\cos^2(x)=1$. As mentioned, students will know and be able to use the Law of Sines and the Law of Cosines. Students will be able to use these laws to help find the area of polygonal regions.

Statistics and probability: Students will enhance their ability to collect, organize, display, and analyze data. Students will understand the statistical concepts of population and sample. Further, they will know and be able to use the concepts of a random sample. They will understand the difference between a sample survey and a census. They will know what bias is and they will know several ways that bias may occur. They will be able to draw conclusions about populations from samples. They will be able to create some types of sampling distributions. They will understand the influence sample size has on statistical conclusions. They will understand, be able to use, and be able to construct confidence interval for proportions using 90% box plots. They will understand the term margin of error as it relates to confidence intervals and sample size.

Students will deepen their understanding of statistical variation. They will know, be able to compute, be able to estimate from a histogram, and be able to use the concept of standard deviation for a data set. (They will have used a computational formula for standard deviation and they will be able to compute the standard deviation using the calculator.) They will know and be able to use the effect a linear transformation of the data has on the standard deviation. They will know that the standard deviation is sensitive to extreme values. They will know the shape of the normal distribution and will be able to use the facts that approximately 68% of the data lies within one standard deviation of the mean, approximately 95% of the data lies within two standard deviations of the mean, and approximately 99.7% of the data lies within three standard deviations of the mean, if the distribution is normal. They will know the relationship between the median and the mean of a normal distribution. Students will compute z-scores and use a table of values to determine the proportion of a distribution over specific intervals. Students will be able to investigate data over time (measurements of the same phenomenon at different points of time). They will use control charts to determine when processes are out-of-control. Students will analyze tests to measure when a process is out-of control. They will understand and be able to use the addition rule for the probability of mutually exclusive events. They will understand factorial notation.

Discrete mathematics: Students will be able to compare and contrast methods of determining public opinion by voting, surveys, and censuses. With voting, they will know and be able to use the concepts of majority, plurality, points-for-preference (Borda), pairwise comparison (Condorcet), sequential-elimination, and approval voting methods. They will understand insincere (strategic) voting. They will know Arrow's Theorem which states that there is no voting method that will always yield a "fair" decision when there are more than two candidates (choices). (The term "fair" is defined.) Students will be able to model situations which involve change over time using sequences. They will know what a discrete dynamical system is. They will understand and be able to use standard subscript notation for sequences. They will understand and be able to work with sequences that are defined recursively and sequences given in closed-forms. They will be able to recognize arithmetic and geometric sequences and series in terms of recursive relations, closed forms, and graphical representations. They will understand the connection between arithmetic sequences and linear models as well as geometric sequences and exponential models. Also, they will know and be able to work with sequences of the form $A_n=rA_{n-1} + b$. They will know and be able to use finite differences to find closed form formulas for some sequences and recurrence relations whose closed forms are polynomial expressions. They will be able to describe the long-term behavior of several sequences. Students will be able to iterate functions both numerically using a graphing calculator and graphically (using the line y=x). They will understand what a fixed point is. They will have several strategies for finding fixed points. They will know what attracting fixed points, repelling fixed points, and a cycle are and be able to determine these behaviors experimentally, and for iterated linear functions, using analytic methods.

Course 4:

The Core-Plus Mathematics Project has completed development and field testing of a flexible fourth-year course continuing the preparation of students for college mathematics. Course 4 consists of four core units for all college-bound students plus sequences of specialized units based on intended undergraduate major. (http://www.wmich.edu/cpmp/course4.html).