Introduction

Beginning in 1989, the National Council of Teachers of Mathematics (NCTM) set forth a vision for school mathematics intended to influence the way teachers taught and students learned mathematics for the ensuing decade. *The Curriculum and Evaluation Standards for School Mathematics* document was followed by publication of the *Professional Standards for School Mathematics* and *Assessment Standards for School Mathematics* [1]. Together these documents contain suggestions that both overtly and subtly alter the mathematical experience for students in grades K-12. In the classroom, the vision includes changes in both teacher culture and student culture by suggesting changes in content, pedagogy, student assessment, and the ways in which various technologies are utilized. While an update of the *Standards* documents is now underway at NCTM, it is anticipated that the fundamental messages and vision will be the same, and will be based in part on what we have learned so far.

The *Standards* documents are not prescriptive. Rather, they give us ideas to think about relative to the areas of teaching and learning mentioned above so that we might improve the mathematical experience of our students. There is much additional documentation to suggest that mathematics education was not fulfilling its role during the 1980's. For instance, see the booklet *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*[2] and references contained in its bibliography.

Shortly after publication of the first *Standards* document, the National Science Foundation (NSF) solicited proposals through a series of Requests for Proposals (RFPs) for new instructional materials development aimed at the improvement of K-12 mathematics education. Several proposals were funded at each of the three school levels: elementary, middle, and secondary.

This booklet contains descriptions of content for five secondary school curricula developed with support of the NSF; five curricula that take us from an abstract vision in the *Standards* documents to concrete reality - five projects which accepted the challenge on page 12 of the first *Standards* document “to reflect on [the *Standards*] vision. Consider what needs to be done and what you can do, and collaborate with others to implement the standards for the benefit of our students, as well as for our social and economic future.” These curricula were selected for this monograph on secondary education

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[1]National Council of Teachers of Mathematics, Reston, Va., 1989, 1991, and 1995 respectively. These publications will be referred to in this monograph as the *Standards* documents. Un-italicized use of the word "standards" refers to an actual standard for school mathematics. There is a list of standards in the *Standards* document. The definition of a standard, which is given on page 2 of the first *Standards* document, is “a statement that can be used to judge the quality of mathematics curriculum or methods of evaluation. Thus, a standard is a statement about what is valued.”

[2]Mathematical Sciences Education Board, National Research Council, Washington D.C., National Academy Press, 1989.

because they constitute the multi-grade, comprehensive, secondary curricula developed with NSF support during this time. Each curriculum described herein has undergone an extensive program of design, pilot testing, redesign, field-testing, evaluation, and redesign before commercial publication. Indeed, each curriculum has been in development for a minimum of five years; a time frame that has allowed the developers an opportunity to think long and hard about standards for school mathematics and the latest research on how students learn. They have had the opportunity to create and evaluate national models of mathematics curricula intended for *all* students about to enter the twenty-first century. The development team for each curriculum included professional teachers, mathematics educators, educational researchers, and mathematicians.

These instructional materials offer distinctive answers to the question: what should a school student know and be able to do as a result of completing a three- or four-year study of mathematics in high school? Indeed, while each curriculum development project knew of the existence of the other projects, and, in fact, freely shared information with the other projects, these five projects did develop independently and they provide five distinct models of innovative curriculum.

Nonetheless, the projects *do* share some common characteristics. As mentioned above, all the projects are multi-grade, comprehensive curricula. That means that the student books and supporting materials span several years of secondary school mathematics[3] and each year of each curriculum contains material for a full year course. In addition, each curriculum includes: at least some content that, in all probability, will be new to many current teachers; pedagogy that focuses on students as active learners; the utilization of technology that requires at least a graphing calculator; and student assessment strategies that significantly augment the ways in which student success was traditionally measured.

All of the curricula are *integrated* curricula. Broadly speaking, that means that each year contains content from several sub-disciplines of mathematics. In our analysis, we describe curricular content using four content "strands": algebra/number, geometry/trigonometry, probability/data analysis/statistics, and discrete mathematics. Following our review of the *Core-Plus* curriculum, we decided to include a fifth strand, mathematical reasoning/logic in our descriptions. (We hope to revise the *Core-Plus* description to include this strand in the near future.) The concept of integration can be implemented in more than one way. Integration could be implemented in such a way that topics from the mathematical disciplines are developed in parallel, in tandem, or side by side, with the topics from the disciplines alternating but remaining quite separate. There have been textbooks in existence for a number of years that have taken that approach. On the other hand, integration could mean that the topics are mixed, blended, combined, or interwoven. What did we find with these curricula? Both types of integration occur. There are occasions where one or more of the curricula reviewed in this monograph will develop some content in a *purely* geometric or *purely* algebraic, etc. manner. But, it is accurate to say that the vast majority of each curriculum can be characterized as a blending together of topics from at least two,

[3]See the included descriptions to determine the number of years each curriculum spans.

and often more than two, content strands. When this latter approach is taken sensibly (i.e. with good reason, as is done in these cases) and smoothly, it can provide several significant benefits. First, it can emphasize the *unity of mathematics* as a single discipline. Second, it can provide opportunities to *do things that were not possible before*. Third, it can provide synergistic *circumstances for the development of two or more topics, enhancing the development of each*. And, of course, it can provide opportunities for review and extension. We saw many examples of each benefit. Moreover, often this blending was accomplished so effectively that at times it was almost self-defeating to separate content into our strands or to consistently list topics under only one strand. (Indeed, in several cases we could have included a topic or concept, such as matrices, under more than one strand.) So, we must emphasize that our review is often a dissection of the curriculum that obfuscates the value of the integration within the instructional materials themselves.

These curricula can also be described as *context-rich*. As implemented here, there is a difference between a *context* and an *application*. Whereas an application is intended to stress the utility of mathematics and thereby increase the value of mathematics for the student, the purpose of an effective context is to heighten the *understanding* of the mathematics. A context, then, is an environment in which the mathematics can be developed or augmented. It is in part a "motivator" and in part a framework, or local structure, upon which to secure concepts and study them. Contexts can be real world situations, abstract mathematical settings, or fantasy. Both contexts and applications are important and valuable, but there is a difference. We found that these curricula included both contexts, in the sense we have described, and applications. It should also be pointed out that contexts and applications are primary vehicles for asking higher-order questions. An emphasis on higher-order thinking is a fundamental objective of these curriculum projects. [Also, see the section on "sense-making" below.]

All the curricula involve the use of technology. Different curricula use different types of technology, although all minimally require the use of a graphing calculator. [See the individual reviews for descriptions of the technology required or recommended for each curriculum.]

One of the most significant characteristics of these curricula is their attention to what we call "sense-making" standards set forth by NCTM: mathematics as problem solving, as communication, and as reasoning. In our opinion it is critical for students to view mathematics as a way of thinking, a way of making sense out of a situation---real or abstract. Such a process ideally taps and reinforces the fundamental human characteristics of wonder and curiosity that seek explanations for conclusions. A fundamental student understanding should be that mathematics involves figuring out why or understanding how something works. Reflecting further on how mathematicians *do* mathematics, it is clear that there are a number of important aspects to the mathematical thinking process. Such a process is much broader than simply providing the technical logical details of the result. The process for a professional mathematician entails, among other things, *exploring examples, searching for patterns, drawing analogies, utilizing intuition, using inductive reasoning, constructing and testing conjectures, justifying special cases, calling upon mathematics background usually from many sub-disciplines in mathematics or other sources, specifying assumptions, creating mathematical* *models, and, finally, providing a deductive argument to substantiate general conclusions.* The curricula in this booklet are filled with many of these attributes of mathematical reasoning.

The curricula differ from each other in some instances in their choice of content, the order in which content is encountered, and in emphasis placed on topics. They differ in style, to some degree in pedagogical and assessment strategies, in the amount and type of technology utilized, and in aspects of the mathematical process they choose to highlight. Our review may help to elucidate some of these differences. However, full appreciation of the differences requires some careful study of the materials themselves.

It is important for the reader to keep in mind that what is offered within the various reviews that follow is *primarily* a description of the content aspects of each curriculum. It was our intention only to provide an answer to the question: *what will a student know and be able to do as a result of completing each of these curricula?*

A curriculum, and mathematics, is much more than content. The mathematical thinking processes that students are developing as they progress are extremely important. Moreover, how students learn is critical to what they learn, and how learning is measured is central to the teaching and learning endeavor. Although there is brief mention of philosophy, pedagogy, and assessment in each of the descriptions we present, much of these facets, as well as the effective integration of topics, the richness of contexts and applications, and the attention to the mathematical thinking process, has been stripped away from the discussion in order to concentrate on the central focus of our investigation.

Our review is based on a combination of published and pre-publication material that was available at the time of our review. We have indicated in our review of each curriculum which part was commercially available at the time of review. The reader should keep in mind that pre-publication material is subject to additional revision before it reaches the commercial market. While it is not anticipated that great changes will be made, some subsequent changes may affect the completeness of descriptions contained in this document.

What *is* obvious from the descriptions contained herein is the rich nature of the mathematical content that will be available for all secondary school students in the twenty-first century. Beginning in 1989, NCTM *Standards* documents presented a vision for school mathematics. The projects that have culminated with the publication of these instructional materials certify that such a vision can, indeed, become part of a reality in which standards are raised, mathematical understanding is increased, and the quality of mathematical experiences for our nation’s students is significantly improved.

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