MATH *Connections*:
A Secondary Core Curriculum

Level: 9-11

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Publisher: IT=s ABOUT TIME

Review Materials: Field-test materials for all three grades were reviewed. A Teacher Resource Package will be available and was reviewed. The package includes a Teacher Edition for each grade, which contains student pages, Teacher Commentary, suggestions about pacing, blackline masters, and solutions to all problems. The package also contains ABooks in Brief,@ which describe the role present materials and its relation to past and to future material. Also included in the package is a set of student assessments. A second set of assessments is also available. Ancillary materials are under development including booklets with challenging problems and projects.

Format/Description: This is a complete three-year secondary school mathematics curriculum intended for all students. The curriculum is integrated; meaning that material from several areas of mathematics are included in any given year. For our purposes, we have centered our description around six strands: **algebra/number/function, geometry, trigonometry, probability and statistics, discrete mathematics and logic and reasoning.** Not all strands appear in each year. The student material is *content-centered*. That is, mathematical topics take center stage in each section usually followed by concepts applied or *connected* to various contextual and Areal-world@ situations. Each grade level is divided into two books, *a.* and *b*, for ease of using; especially with block scheduling. The *a.* section is meant to precede the *b.* section. Each section is divided into chapters and each chapter is divided into several sub-sections. Each sub-section begins with stated learning objectives for that sub-section and several student activities within explorations followed by a problem set. The activities are coded with icons indicating either a discussion topic, a writing topic, or an activity that should be done before proceeding. Some sub-sections contain ideas for longer student projects. The margins of the student materials contain **Thinking Tips**, **About Symbols,** and **About Words** (notes that detail how some everyday words have more specific meanings in mathematics). Appendices for each level detail technology information helping students learn to use a TI-82 (83) Graphing Calculator, use a spreadsheet, program a TI-82 (83), etc.

The Teacher Commentary includes an overview and general comments on the student materials as well as suggestions concerning the implementation of the material in the classroom. It contains comments and answers to the activities and problems contained in each sub-section. Blackline Masters are included, also.

Pedagogy: The curriculum delivery in the classroom is based on directed student activities where students Atry out@ approaches suggested by the materials or apply concepts with which they are working. There are follow-up homework problems. The classroom activities may be directed by the teacher using the text along with a more conventional teaching approach or students can read the text and do activities on their own with the teacher acting as both a guide and provider of additional material or explanation. These materials have been implemented with students working individually as well as with students working in groups. Sometimes the Teacher Commentary or student materials suggest student interaction such as comparing answers to activities or general discussion. Other times, choices are left to the teacher.

Technology: Graphing calculators are essential for the delivery of the curriculum. Such calculators are used in the majority of the material. The TI-82 (83) series is supported by the student materials. In addition, spreadsheet capabilities are used in some units. For example, spreadsheets are used in the material on linear programming. (The use of Excel is supported in an appendix to that material.) In addition, students will use technology to produce scatterplots, fit regression lines, graph functions, observe patterns, and do tedious calculations. Some programming on graphing calculators is included. The availability of a dynamic geometry program such as the *Geometer=s Sketchpad* is suggested for some of the geometry material.

Assessment: Assessments are provided for each sub-section of each book. These assessments include Aquizzes@ for each chapter and end-of-section assessments.

Content Overview: **MATH** *Connections* is an integrated curriculum in the sense that content associated with algebra, geometry, and data analysis appears at each level. (Note that some coordinate geometry regarding straight lines appears in the first year of the curriculum and is listed under the **algebra/number/function** section for Year 1 below.) In addition, this review identifies several strands that occur in the curriculum taken as a whole: algebra/number/function, geometry, trigonometry, probability and statistics, discrete mathematics and logic and reasoning. Not all strands appear in each year. See below for details concerning the appearance of these strands in the various years. It may be noted that Year 1 material is heavily concentrated in algebra, Year 2 material is heavily concentrated in geometry, and Year 3 contains considerable material in pre-calculus and discrete mathematics. While the above emphases suggest a more traditional division of material, it must be noted that many of the concepts within these topics are non-traditional. Moreover, Atraditional@ topics associated with these emphases may appear elsewhere in the curriculum. For example, axiomatic proof involving geometry is covered in Year 3 despite the emphasis on geometry in Year 2. As a second example, while the concept of function may be more traditionally included in a course on pre-calculus, the topic of functions appears throughout the entire **MATH** *Connections* curriculum. In the words of the developers, **"a major goal of the curriculum is the development of higher-order thinking skills."** The developmental approach most often used is for the students to actively explore a concept in order to develop experimental evidence and/or recognize a pattern, be assured the pattern is accurate, represent the pattern by a symbolic formula, and then apply the formula in real-world situations. There is a strong symbolic component throughout the curriculum. It is the philosophy of the developers that a great deal of mathematical power comes from being able to Aunderstand formulas and be able to transfer use them to a variety of contexts.@ Graphical and tabular representations are also included throughout. Many times students are asked to think about and discuss or write about what they are doing and why they are doing it. The problem set reinforces and extends, to some degree, the material in the sub-section. It usually does not contain traditional simple drill and practice problems. It often does contain additional material and/or cases not covered in the preceding narrative as well as applications. Our description of what students completing this curriculum will know and be able to do is given in terms of the strands mentioned above. Some topics, such as coordinate geometry, could be listed under more than one strand. However, when topics are closely integrated, the decision has been made to list them only under one strand in each year. Moreover, the description below does not necessarily describe the order in which topics are encountered in the curriculum within each year. For the most part, topics which are mentioned or worked with briefly but not emphasized to any degree are omitted. However, in some places we say, simply, that students are Aexposed@ to such topics.

Year 1

Algebra/number/function: Using appropriate variables and constants, students will be able to write an equation to represent one quantity in terms of one or more other quantities. They will be able to evaluate equations or formulas, including exponential equations, for specified values. Given a table of values, they will look for any patterns and, if found, write a formula to represent a generalization of the pattern. They will be able to use the correct order of operations to compute an arithmetic expression or to simplify an algebraic expression. Students will be able to solve an equation in one variable, by applying the appropriate laws of algebra, including the commutative and associative laws of addition and multiplication and the distributive law. They will be able to write an exponential expression to represent a real-world situation involving exponential growth or decay. Students will be able to construct a rectangular coordinate system and plot points that represent ordered pairs of coordinates. Given the coordinates of two points, they will be able to compute the slope of a line. They will recognize that a vertical line has no slope. They will be able to write a linear equation in the form y=mx+b when given two points or the slope and y-intercept. Using a graphing calculator, students will investigate the effect of a change in a or b on the graph of a linear equation y=ax+b. They will also explore the effect of integral changes in integral exponents on the graph of $y=x^a$, for a$1. They will be able to graph a linear equation in the form y=mx+b (with and without the aid of a calculator). They will compare graphs of linear equations in terms of their slopes. Given a real-world situation, they will be able to write a linear equation of the form y=mx+b, draw its graph, and use both in the analysis of the problem. Using a tree diagram to organize possible outcomes, students will develop an algorithm for an efficient way to play a number guessing game and apply it in solving real-world problems. They will graph inequalities in one variable on a number line. Students will utilize graphs and/or tables of values to compare two sets of data. By plotting points, they will graph two linear equations on the same set of axes, find the point of intersection, and make comparisons. They will also use the calculator to graph pairs of equations (some non-linear) and find their point(s) of intersection. They will solve systems of two linear equations of the form y=mx+b algebraically (substitution method). They will graph equations of the form ax+by=c by plotting points (including the x- and y-intercepts) and solve systems of these equations graphically. Students will be introduced to a linear programming problem and its terminology (e.g. constraints, region of feasible solutions). They will graph a system of linear inequalities and investigate where maximum profit occurs within the feasible region.

Students will be able to determine whether or not a situation describes a function. They will be able to identify the domain and range of many functions, use function notation, and evaluate a function at specified values. They will write a formula for a sequence of numbers and use it to find the n^{th} term in the sequence. They will use a spreadsheet to generate a sequence. They will determine whether or not two functions are equal on the domain of counting numbers. Students will be able to make a table and/or graph to represent a function, including step functions. They will write a function and create a simple program for the calculator to convert temperature scales (Celsius to Fahrenheit). They will write a linear or exponential growth (involving compound interest or population growth) function for real-world situations and graph it on the coordinate plane. They will also graph a function on the calculator and use the trace feature to evaluate it at specific values. Given two or more functions, students will be able to find and evaluate a composite function. They will be able to write a function as the composite of two or more simpler functions. They will be able to describe some real-world processes as composite functions. They will examine the laws of commutativity and associativity with respect to the operation of composition (e.g. fBg...gBf).

Students will be able to compute the absolute value of a number. They will be able to calculate a number written in exponential form. They will recognize the power of exponents by exploring the classic doubling problem (1 penny, 2 pennies, 4 pennies, 8 pennies, etc.). They will be able to convert numbers expressed in scientific notation to standard notation and vice versa. Using the calculator, they will solve problems involving operations with scientific notation. By investigating patterns in multiplying integers, they will draw conclusions concerning the multiplication of negative integers.

Probability and statistics: Students will recognize the need for a systematic approach to counting and they will investigate and develop a variety of strategies to solve counting problems in several contexts. They will be introduced to set notation and terminology (e.g. intersection, union, disjoint, empty set). They will be able to list the elements of a set, count the number of elements, and identify subsets. They will realize that A1@ and Ac@ are associative operations. Students will use Venn diagrams (2 and 3 circles, with a known quantity for the intersection) to partition sets into disjoint subsets and determine the number of elements in these subsets. They will use tree diagrams to find all possible outcomes for an event. In cases that involve two possible outcomes at each level, they will become aware of the symmetry of the branches and the need to create only a partial tree to ascertain the total number of outcomes. They will be able to apply the Fundamental Principle of Counting. Students will utilize these counting strategies in assigning probabilities to events on the basis of intuition, past experience, experimentation, or theory. They will recognize that a probability must be a number between 0 and 1, inclusive. They will be able to identify certain and impossible events, assigning an appropriate probability to each. Using the definition of probability for events involving equally likely outcomes (i.e. P(E)=# of acceptable or favorable ways/total # or possible ways), students will be able to find the probability of an event. They will become familiar with expectation as they compute the expected frequency of an outcome for a given number of repetitions of an experiment. They will be introduced to conditional probability (not stated as such) as they find the probability of an event given a known previous occurrence or condition. They will be able to find the probability of the complement of an event as well as the intersection or union of two events. They will examine the generalization P(AcB)=P(A)+P(B)-P(A1B). They will apply their knowledge of probability in finding medical indices that could serve as predictors of survival in trauma cases. Students will explore simulation as a means of modeling real-world situations (e.g. waiting times at the post office). They will become familiar with random number tables and use the calculator to random generator feature to simulate the flipping of a coin N times.

Students will investigate sets of data that have been gathered by them or another source in terms of measures of central tendency and spread of data. They will be able to compute the mean, median, and mode (using a calculator with statistical capabilities, if they wish) and determine which is the most appropriate measure of the center. They will analyze the effect of a change in one or more data values (including an extreme value) on the mean, median, and mode. Given a set of data, they will be able to construct a bar graph, a dot plot (line plot), a stem-and-leaf plot, and a histogram. Students will be able to find the range and mean absolute deviation for a set of data. They will be able to determine the five number summary for a set of data, construct a boxplot from these figures, and analyze the spread of the data. They will be able to compare and contrast sets of data represented by two or more boxplots, histograms, or back-to-back stem-and-leaf plots. They will use a spreadsheet to calculate the mean, mean absolute deviation, and the five number summary for a set of data. They will explore and calculate the variance and standard deviation for a set of data. They will compare and contrast two sets of data, draw conclusions based on an analysis of measures of central tendency and spread of data, and justify their conclusions. Students will be able to construct a scatterplot from a table of values and use it in the analysis of the data to observe any overall trends and estimate data. They will determine when a table or a scatterplot might be the better representation for a set of data. They will also use linear interpolation and the least-squares (regression) line to estimate data. In addition, they will use linear models for extrapolation (prediction). Using a calculator, they will be able to find the least-squares line, use it to make predictions, and determine whether or not their estimates are reasonable.

Year 2:

Algebra/number/function: Students will recognize equations that can be considered identities. Furthermore, they will use area models to illustrate the distributive property and, in particular, justify the identity that $(x+y)^2=x^2+2xy+y^2$. They will explore situations where one variable is directly proportional to another (y=kx) and situations where one variable varies as the square of another ($y=kx^2$). Inverse variation is also explored. They will find the constant of proportionality, write an equation, and use it to find additional information. Students will solve for a missing term in a proportion using the technique of cross-multiplying. They will be able to solve equations involving the Pythagorean Theorem and use a calculator to find the square root of a number, when needed. Students will understand the meaning of compound inequalities such as $2<x<5$ and be able to separate them into two inequalities and vice versa. They will explore functions defined by multiple rules such as piecewise linear and step functions. They will have experience describing real situations with three variables and identify independent and dependent variables. In particular, they will explore real-valued functions of two variables. With tables and formulas. They will be familiar with notation such as T(x,y) to represent the value of a function with independent variables x and y. They will have a strategy for envisioning such relationships involving three variables using a 2-dimensional graph instead of a 3-dimensional graph, especially when the variables are discrete. The strategy involves plotting A contour lines@ of the graph determined by fixing one of the variables at various points and graphing the resulting relation. Using complete graphical information obtained from this process, for example, students will be able to determine whether or not the function of two variables is linear. They will know what to look for symbolically to determine whether or not a function of any number of variables is linear. Also, here, a *matrix* is defined as a function of two variables where the independent variables represent values in the set {1,2,..,*n*} for some particular *n*. Students will be able to distinguish between categorical and measurement variables. They will have experience making some theoretical models and comparing the model to actual data. They will understand the term Aresidual@ as the actual value minus the predicted value. They will explore Aadditive@ models of functions versus Ainteractive@ models of functions of two variables (where there is usually a term containing the product of the two independent variables.)

Students will be able to identify linear equations in two variables in several symbolic forms such as y=2x+1 and 2x+3y=6. They will be able to solve for one variable in terms of another in these situations. They will have experience solving systems in two unknowns using elimination. In fact, they will be able to work with the Gaussian Elimination algorithm and back substitution to solve systems of liner equations involving up to three variables. They will be able to represent such linear systems using matrices and be able to apply Gaussian elimination using row operations both by hand and using technology. They will be able to determine when this process reveals a system which has no solution, infinitely many solutions, and a unique solution. They will be able to interpret the latter situation geometrically with two and three unknowns. Students will use these concepts in many applied settings. For instance, within mathematics, they will have experience Afitting@ an equation of the form $(x-a)^2+(y-b)^2=c$ to three data points in the plane by solving a system of linear equations in three unknowns using Gaussian Elimination and back substitution. By graphing the equation, they will see this as determining a circle through the three points. They will also have experience fitting a parabola to *some* sets of three points using elimination. Students will also be able to use matrices for organizing, sorting, and manipulating information. They will have contextual reasons for defining the addition, subtraction, and multiplication of matrices, and know when these operations can be performed. They will have seen and worked with some abstract properties of operations with matrices, e.g. commutativity of matrix addition, but not multiplication; associativity of addition and multiplication; and properties of the "zero" matrix (all entries are zero) with addition and identity matrices (ones along the diagonal and zeros otherwise) in matrix multiplication. In addition, they will understand and be able to use the concept of multiplying a matrix by a scalar. Students will have experience working with the least squares computations by hand and determining the Acoefficient of determination@ (usually called r^2). They will be introduced to Σ-notation. They will know the basic shapes of quadratic polynomials and at least the shape of the cubic $y=x^3$. They will see what a, b, and c do in each of the cases $y=ax^2$; $y=x^2+b$; $y=(x+c)^2$; and $y=x^2+cx$, for *at least* n=2.

Geometry: Students will realize the advantages and disadvantages of modeling real-world situations; in particular, they will distinguish between shortest distances on a map vs. a globe. They will be able to identify units of measure for length and time, measure lengths using standard and nonstandard units, and recognize the importance of standard units of measure. Students will be able to compute the perimeter of equilateral polygons using the formula P=ns, as well as perimeters of other regions (e.g. rectangles, triangles). They will identify polygons and name their vertices, sides, and angles. Using a compass and straightedge, they will construct the perpendicular bisector of a line segment and the perpendicular from a line to a point not on the line. They will also be able to construct a triangle, given its side measurements, and they will be introduced to the triangle inequality when they are given measurements that cannot form a triangle. They will construct an isosceles triangle, as well as the bisector of its vertex angle. They will be able to determine the axes of symmetry for a figure, including an equilateral triangle, rhombus, rectangle, square, and other regular polygons. They will investigate the property that the diagonals of a rhombus are perpendicular bisectors of each other. They will classify certain quadrilaterals (parallelogram, rhombus, rectangle, square) according to sides and angles. Students will investigate strategies for finding the area of a region, including the use of formulas for a rectangle, a triangle and a parallelogram, Atiling@ a region and counting unit squares, and the Adivide and conquer@ algorithm which divides a region into smaller areas that can be found and summed to yield total area. Moreover, they will realize that the area of a polygon can be determined by triangulating it and adding together the areas of the triangles. They will identify the Pythagorean Theorem, use it to find the length of a side of a right triangle when the other two sides are given, and use its converse to ascertain whether or not a triangle is a right triangle. They will convert some English units of measure (e.g. square inches to square feet) and some metric measures (e.g. square centimeters to square decimeters). They will be able to determine the volume of a rectangular prism.

Students will investigate the concept of proportionality. They will be able to use a scaling factor to determine measurements of similar figures and create a proportional drawing. Given measurements of two similar figures, they will find the scaling factor. They will conclude whether or not two triangles are similar by comparing lengths of corresponding sides and finding the ratio of similarity, if possible. Students will identify kinds of angles (acute, obtuse, right, straight, reflex) and examine different ways to measure them using slope, degrees, and AA-measure@ (where the class decides on a unit of angle measure). They will convert slope measure of an angle into degree measure and vice versa, using the $\tan^{-1}$ and tan calculator keys. They will realize that angle measurement is unaffected by scaling. They will compute and compare perimeters and areas of scaled figures (i.e. if a figure is scaled by a factor of k, its area will change by a factor of k^2). They will also compute the volume of a scaled rectangular prism and realize that its volume changes by a factor of k^3. Students will know that the measures of vertical angles are equal and that supplementary angles can form a straight angle. They will explore corresponding angles and alternate interior angles formed by two lines crossed by a transversal. In particular, they will see that two lines cut by a transversal are parallel if and only if the measures of a pair of corresponding angles (or alternate interior angles) are equal. [Software such as the *Geometer=s Sketchpad* is suggested in the Teacher Commentary as a means to explore angle measures.] Students will be guided through a series of steps leading to the fact that the sum of the angles of any triangle is 180E. They will explore the sum of the measures of the interior angles of any polygon to eventually discover the generalization (n-2)180E for an n-gon. They will realize that the sum of the measures of the exterior angles of any polygon is 360E. Furthermore, using appropriate formulas, they will compute the measure of a single interior angle and exterior angle of a regular polygon. Focusing on triangles, students will investigate the congruence principles (SSS, SAS, ASA, AAS) and use them to determine a triangle from given angle or side measurements. They will recognize that SSA and AAA are not valid principles for determining a triangle.

Students will be able to construct regular polygons with a given number of sides with geometry software such as the *Geometer=s Sketchpad*. They will see many properties of the circle. They will know its Acenter-radius@ definition (i.e. the set of points in the plane a fixed distance from a given point). They will recognize that every diameter of a circle is an axis of symmetry. They will know the definition of rotational symmetry and be able to compare the rotational symmetry of regular polygons to the rotational symmetry of the circle. They will know that the set of circles that pass through two given points have centers on the perpendicular bisector of the segment joining the points and conversely. They will be able to construct a circle passing through three points with straight edge and compass by locating its center. They will be able to represent a circle parametrically using such parametric representations as $(\cos\theta, \sin\theta)$ for the unit circle. They will investigate what a transformation $(a+r\cos\theta, b+r\sin\theta)$ does to the unit circle. They will be able to determine parametric representations for circles in the plane and be able to utilize such representation along with technology that plots parametrically (such as the TI-82) to display designs created from circular arcs. Using results on changes in area and perimeter of similar figures, they will explore relationships between the area and perimeter of a circle. For example, they will compare the area and perimeter of a circle to the area and perimeter, respectively, of the unit circle. They will discover that the area of the unit circle is π (square units) and develop several approximations to its value using geometric means. They will see that the area of a circle can be found by multiplying its circumference by its radius and dividing by two. They will also express the area and perimeter of a circle as a function of its radius (i.e. $A(r) = \pi r^2$ and $C(r)=2\pi r$). They will be able to use the proportional relationship between a central angle of a circle and the length of the corresponding arc and the proportional relationship between a central angle of a circle and the area of the corresponding segment of the circle. They will be able to supply justification for special cases. They will also be able to use the relationship between an inscribed angle and one of the central angles with the same endpoints as the inscribed angle and will have worked through a justification of this relationship. They will know that some properties *don=t* characterize the circle. For example, they will know that there are curves other than the circle which have constant width (e.g. a Reuleaux Triangle). Students will investigate three-dimensional shapes. They will have two-dimensional strategies for describing these shapes (such as a cone). They will be able to use *nets* (two-dimensional patterns) that fold into polyhedra and other solid shapes. They will determine a method, using the Atwo-dimensional@ Pythagorean Theorem for calculating the height of a pyramid with equilateral triangular sides and a regular polygon as a base knowing the length of an edge. They will realize that a pyramid with a regular polygon as base and equilateral triangular sides must have a base with fewer than six sides. They will realize that a regular polyhedron has the property that the sum of the angle measures at each vertex is less than 360^0. They should have developed a justification for the fact that there are only five regular polyhedra. They will be able to use two-dimensional contour curves to describe irregular shapes such as geological landscapes. They will be able to describe a cone using a net formed by removing a sector of a circle. They will be able to calculate the radius of the base of a cone as a function of the central angle measure of the sector of the circle removed. They will be able to calculate the height of a cone as a function of the radius of the base of the cone. They will be able to compose these functions to represent the height of a cone as a function of the central angle of the central angle measure in the net. They will be able to describe solids such as prisms and cylinders using two-dimensional cross-sections. They will be able to calculate the surface area of a prism from information about its height, the number of edges of its base, and the length of each of those edges. They will provide some detail in the development of a formula for the volume of a cone using mostly geometric arguments. They will know Cavalieri=s Principle and have worked through a plausibility argument by comparing cross-sectional slices of two prisms of the same height where one is a right prism and the other is oblique and letting the thickness of these cross-sectional slices shrink.

They will develop a very intuitive notion of this latter explanation as a Alimiting A procedure. They will use Cavalieri=s Principle to determine the volume of solids. For example, they will have seen the Principle used, with some algebraic manipulations, to develop a formula for the volume of a sphere. They will investigate solids of revolution (without coordinates) of many bounded plane regions around a line. They will have seen evidence based on geometry and algebra to support the Pappus-Guildin Theorem which states that the volume of a solid of revolution is the product of the area of the region and the distance traveled by the center of gravity (centroid) of the region. They will explore the center of gravity of triangles, rectangles, circles, and semi-circles and, thus, be able to determine the volume of the solid obtained by rotating one of these regions about a line (which does not intersect the region). They will be introduced the a coordinatization of three dimensions using the three standard axes in the standard orientation. They will have seen the formula for the distance between to points in the plane and two points in three-space developed. They will be introduced to Aset builder@ notation. They will be able to describe many three-dimensional objects using sets of triples satisfying algebraic equalities and/or inequalities. They will know the equations for a (two-dimensional) circle of radius r and a (three-dimensional) sphere of radius r. They will see that the three axes do not need to represent spacial measures. They will see examples where solids can be used to convey information when one of the axes measures another quantity such as time.

Trigonometry: Building on the concept of similarity of right triangles, students will be introduced to the trigonometric functions (sinθ, cosθ, tanθ, cscθ, secθ, cotθ, where θ is an acute angle) as ratios of the sides of a right triangle. They will use the sine, cosine, and tangent functions to find the length of a side of a right triangle. They will find the measure of an acute angle of a right triangle, using the inverse trigonometric functions ($\sin^{-1}\theta$, $\cos^{-1}\theta$, $\tan^{-1}\theta$). They will generalize a formula for the area of a parallelogram in terms of its sides and an acute angle (either interior or exterior): Area=absinA. Students will follow developments of the Law of Sines and the Law of Cosines (for acute angles only) and use them to find the length of a side of a triangle. Through specific questioning, they will explore a proof that Athe angle made by a tangent line and the radius at the point of tangency is a right angle.@ They will calculate $(\sin\theta)^2+(\cos\theta)^2$ for several angles, make a conjecture, and follow a generalization to the conclusion that $\sin^2\theta+\cos^2\theta=1$. They will also see that $\tan^2\theta+1=\sec^2\theta$. Given tables of sine and cosine values for $0E<\theta\leq 50E$, where θ is an integer, students will compare values. Using a diagram of a right triangle with sides a, b, c and angle θ, they will explain identities such as sin(90E-θ)=cosθ and tan(90E-θ)=cotθ. They will explain the connection between the slope of a line and the tangent of an angle. They will examine the graphs of y=sinθ and y=cosθ on a graphing calculator and be introduced to several real-world examples which could be modeled by the sine and/or cosine curves. They will relate the graph of the unit circle to the sine and cosine of an angle and, hence, extend the domains of the sine and cosine functions.

Probability and statistics: Students will perceive differences between discrete and continuous data. They will enhance their ability to create appropriate representations (e.g. stem-and-leaf plot, boxplot, scatterplot, dot plot (also known as line plot)) for sets of data and compare and contrast these displays. Using primarily a dot plot, they will examine the distribution of data in terms of its symmetry, whether or not it is bell-shaped or bimodal, and possible outliers. They will be introduced to bias when they encounter data that is not Adistributed approximately symmetrically around the true value.@

Logic and reasoning: Students will write the converse of statements and decide whether or not they are true. If true, they will explain why; if not true, they will provide reasons or counterexamples. For example, they will state the converse of AIf two polygons are similar, their corresponding angles are congruent@ and give a counterexample to show that the converse of this statement is false. Students will be introduced to the idea of proof as Aa logical argument that explains why a statement must be true@ and they will be expected to provide logical arguments that justify facts such as Aangles of an equilateral triangle must be equal@ and Aa quadrilateral must be a rhombus if its diagonals are perpendicular bisectors of each other.@

Year 3:

Algebra/number/function: Students will know the concept of function and its representation by sets of ordered pairs, graphs, input/output tables, and symbols. They will recognize the terms dependent and independent variable and the usual functional notation and language, such as Ay is a function of x.@ They will be able to apply the vertical line test to a graph in order to determine whether or not the graph represents functions. They will know the terms *domain* and *range* and be able to find both implicit domains of functions and domains determined by context (e.g the variable length won=t be negative). They will be able to do this work with polynomials, especially quadratics, and functions involving absolute value in real world settings. They will have seen both the general form ($y=ax^2+bx+c$) and @vertex form@ ($y=a(x-h)^2+k$) of quadratic functions and know the term parabola as a description of the shape of the graph of a quadratic function. They will know the formula for the vertex of a parabola. They will know the concept of functional composition and the notation for it. They will be able to compose several functions and consider the question as to whether or not a composition exits. Conversely, they will be able to separate many functions into compositional components. They will know the term *one-to-one* function and be able to apply the horizontal line test to the graph of a function in order to determine whether or not it is one-to-one. They will utilize some compositions to shift (Atransform@) the graph of some functions and/or study the properties of functions. For example, if f(x)= f(-x) for all x, then the graph is symmetric around the y-axis. Similarly, if g(x)=x+b, and f is a polynomial, then students will know that the graph of gBf is a vertical shift of the graph of f. Students will know the term *inverse function* and have strategies to determine the symbolic representation of the inverse of some simple functions when the symbolic representation of the initial function is given. They will know some properties of inverse functions such as the domain of the inverse function is the range of the initial function and the range of the inverse function is the domain of the initial function. They will also know properties such as $f(f^{-1}(x))= x$ and $f^{-1}(f(x))=x$. They will understand and be able to determine the graph of an inverse function from the initial function by reflection of the graph over the line y=x, when the inverse function exists. Similarly, they will be able to create the table of an inverse function by interchanging the values of x, or input, and y, or output, when the inverse exists. They will know that some values of a function are more significant than others relative to describing the behavior of the function. For instance, they will know that maxima and minima are important descriptors, as are Azeros@ of a function. They will be able to connect finding the zeros of a function to solving an equation. They will have several strategies, including guess and check, for finding zeros of some functions. Moreover, they will be able to use the quadratic equation to find the zeros of a quadratic function. They will understand the term discriminant and understand the relation of the discriminant to the number of (distinct) zeros of a function and roots of the corresponding quadratic equation. They will also be able to use factoring to find the roots of an equation in simple cases. Finally, they will be able to use the technology of graphing calculators to find approximations of zeros of a function. They will also realize the limitations of such technology. For example, to use the trace feature of a graphing calculator, it is necessary to have an appropriate viewing window, first. Students will recognize an exponential function symbolically as a Afunction which has a variable as an exponent.@ They will be able to recognize and identify the basic shape of the graph of an exponential growth function and differentiate it from the graph of an exponential decay function. They will be able to determine symbolic representations (in base 2) of exponential situations with respect to doubling time (in growth situations) and halving time (in terms of decay situations). They will understand and have developed the basic laws of integer exponents (including zero) and extended the laws to all real exponents. They will understand the meaning of rational exponents in terms of roots and powers. They will recognize the terms *base* and *power* as applied to an exponential expression. They will explore many applications of exponential models including half-life situations and compound interest. They will develop an intuitive sense of continuous versus discrete situations; where continuous, implies, for example, that a process goes on Avirtually@ without interruption (e.g. bacteria are virtually always growing). They will understand logarithms with base b>0 as the inverse of the exponential function $y=b^x$ and, hence be able to determine its graph, etc. They will see the value of logarithms as functional models in many situations. They will be able to use the concept of logarithms to solve exponential equations and analyze data. For example, they will be able to apply a linear fit to a log plot (a plot of (x, log y)) of data points as a method for determining a functional model which is an exponential function. They will be able to apply a linear fit to a log-log plot (a plot of (log x, log y)) of data points in order to determine a functional model which is a power function. In the latter situation, they will see that the slope of the Afit@ line is the exponent of x in the power function model. (So, they will see at least one purpose for an irrational exponent.) They will know the number *e* intuitively as the Alimiting@ value of the expression $(1+1/n)^{1/n}$ as n gets large. They will know the terms and symbols for common log, symbolized as log, and natural log, symbolized as ln. They will have developed the basic properties of logarithms and see their benefit in many situations. They will be able to determine estimates for some logarithms and use graphing calculators to find approximate values of other logarithms. They will be introduced to the intuitive concept of a series (Maclaurin expansion) and the limit of a series, and see how polynomials can be used to approximate the trigonometric and exponential functions, thus providing a link between these three families of functions.

Geometry: Students will have had a brief historical introduction to axiomatic geometry. They will have seen the Common Notions and Postulates as well as proofs of several Propositions (Theorems) from the *Elements* of Euclid. They will have supplied reasons in the steps of many of Euclid=s Propositions. They will have proved several Propositions themselves. Their historical tour will have included a discussion of Postulate 5 (the AParallel Postulate@) as well as some of Saccheri=s and Riemann=s work in non-euclidean geometry. They will have seen how Postulate 5 connects with basic results concerning parallel lines, parallelograms, the sum of angle measures in a triangle, and the existence of a square. They will have had some experience proving statements in non-euclidean geometry; especially using *Saccheri quadrilaterals*. Some of these proofs depend on diagrams. They will be able to compare the three geometries: Lobachevskian, Riemannian, and Euclidean in terms of axioms about parallel lines and the sums of angle measures in a triangle.

Trigonometry: Students will recognize the graph of a periodic function. They will be able to determine the period of a periodic function from its graph and from a table. They will know that if *a* is the period of a function, then f(x+*a*) = f(x) for all x in the domain of f. They will see examples of periodic functions in many contexts. They will work with both degree and radian measure of angles and be able to convert from one angle measure to the other. Students will understand the definition of sine, cosine and tangent functions as they are related to the unit circle in the coordinate plane. They will be able to recognize the graphs of the sine, cosine and tangent functions they will know the domains (both in terms of angles and radians) and ranges of these three trigonometric functions, what the maxima and minima are with respect to the sine and cosine functions, and where the asymptotes are with respect to the tangent function. They will have seen the transformations cf(x); f(cx); f(x)+b; and f(x+b) and how combinations of these transformations affect the graphs of each of the three trigonometric functions mentioned above. In particular, they will understand how these transformations affect amplitude and period of the sine and cosine functions. They will be able to determine the symbolic representation for a Asine-type@ function given its graph. They will be able to fit a sine function to a periodic data set. They will be introduced to the idea that it is possible to find combinations of trigonometric functions which will make a reasonable model for any periodic situation. (Students will see that this means sometimes modeling a discrete situation by a continuous function.)They will be able to use the inverse sine, cosine and tangent functions. They will recognize the graphs of these functions. They will be able to determine some values of an inverse trigonometric functions based on the definition of the parent trigonometric function and its relation to the unit circle. They will be able to use technology to determine other values.

Probability and Statistics: Students will enhance their ability to use counting techniques such as the Fundamental Counting Principle (sometimes known as the Multiplication Principle) and the technique of partitioning a set into disjoint subsets, counting the objects in each subset, and adding the totals in order to determine the number of objects in the whole set. They will have seen the development of basic counting formulas such as permutation and combination formulas and the general formula for arranging N objects in a set which has the property that it can be partitioned into sets of indistinguishable objects of several types (like the letters of the word Mississippi). They will be able to apply these concepts and techniques both separately and together in counting contexts as well as in the determination of theoretical probabilities involving equally likely outcomes. They will know the concept of conditional probability both in terms of a restricted sample space in the special case of equally likely outcomes and as *defined* by the formula: P(A|B)=P(A1B)/P(B). They will understand the concept of independent events as two events, A and B, which satisfy the formula P(A1B)=P(A)P(B) and be able to determine whether or not events are independent in many situations. They will know the definition of expected value and be able to compute it in many contexts. They will know the definition of binomial probabilities in terms of measuring the probability of success in a sequence of independent experiments. They will have seen the development of binomial formulas for the probability of k successes in N trials. They will know that the expected value in binomial experiments is equal to the given probability of success. They will be able to construct bar graphs of binomial probability distributions and be able to relate certain probabilities to the areas of the rectangles in such bar graphs. They will have seen a formula for the standard deviation in N trails of a binomial experiment with probability of success *p*. They will know that as N increases, the binomial distribution can be approximated by a normal curve whose line of symmetry is the expected value of the binomial situation. They will be able to utilize the (approximate) area under a normal within one, two, and three standard deviations away from the expected value to find probabilities in situations where outcomes are assumed to be normally distributed. They also will be able to utilize the area under the normal curve and its standard deviation to approximate binomial probabilities. They will have seen an analytic formula for the normal curve and they will know that the standard deviation for a normal curve is a measure of how Abunched up@ the curve is around its expected value. They will know what a random sample of a population is and have discussed bias. They will use the normal curve to evaluate results from a sample and know what a 95% confidence interval. They will know how the term Amargin of error@ is related to a 95% confidence interval and be able to compute 95% confidence interval is several situations.

Discrete mathematics: Students will know and be able to use several strategies to solve a variety of optimization problems. In particular, they will know, be able to explain, and be able to use the Greedy Algorithm and Dynamic Programming to solve (weighted) Ablock diagram" problems. They will have strategies to solve two-variable linear programming problems (including mixture and transportation problems, etc.). These strategies include graphical methods and the use of algebra involving Adictionaries@ (which is essentially the simplex method involving slack, basic, and non-basic variables). They will understand the value of technology in solving linear programming problems with many variables and will utilize technology (spreadsheets) to solve some linear programming problems. They will have some experience solving linear programming problems involving more than two variables by hand. They will understand the concept of a *graph* with vertices and edges. They will know the meaning of connected graphs, weighted graphs, trees, spanning trees and cycles. They will understand and be able to use Kruskal=s Algorithm for finding minimal spanning trees in a connected weighted graph. They will have seen two derivations of the basic arithmetic-geometric mean inequality for two positive numbers (i.e. the arithmetic mean is less than or equal to the geometric mean). They will use this inequality in Geometric Programming to solve several optimization problems, many of which are classic calculus problems. Students will be introduced to concepts of infinity. They will explore some paradoxes involving the infinite. They will be introduced to several infinite processes which tend toward a Alimit.@ Such processes include: unending decimal expansions, general (convergent) geometric series, Archimedes= AMethod of Exhaustion,@ and Riemann sums for area under a curve (integration). They will intuitively understand limit in a ABolzano-Weierstrass@ sense. (This term is not used.) That is, students will Aclose down@ on the limit as a number between bounds which get closer and closer tegether. (The concepts of Maclurin expansions and the limit of $(1+1/n)^{1/n}$ as n gets large mentioned above could fit into this context.) Students will be able to work with formulas and algorithms for finding the limit of a convergent geometric series and the fractional representation of a repeating decimal. They will also be able to use this infinite Aclosing down@ process to position (infinite) decimals on the number line. They will know that irrational numbers do not have repeating decimal expansions. Yet, students will have a Afinite@ strategy for positioning the square root of a positive integer on the number line. They will also encounter situations with infinity where there is no limit, such as in the case of the (unending) natural numbers. Students will see the categorization of numbers in terms of natural numbers (positive integers), integers, rational numbers and irrational numbers and investigate how these categories relate to each other. They will recognize the term *dense set* and know, for example, that the set of rational numbers is dense, but the set of integers is not. They will know that both the set of rational numbers and the set of irrational numbers have Agaps@ in them but, taken together to form the real numbers, there are not gaps. They will be introduces to the cardinal concept (term not used) which states that two sets are equivalent if there is a one-to-one correspondence between them. They will see that the rational numbers and the positive integers are equivalent, but that the real numbers and the positive integers are not. Thus, they will be introduced to different Asizes@ of infinity.

Logic and reasoning: Students will understand what an axiomatic system is. They will know the role of undefined terms and the assumed truth values of the axioms. They will know the difference between circular and non-circular definitions and have experience determining Acharacteristic properties@ which define an object. They will have experienced the danger of hidden assumptions in situations. They will understand what a *consistent* axiomatic system is. They will know the method of determining consistency by finding an Ainstance@ (realization) of the system. They will understand that one can prove inconsistency by providing a logical contradiction within the system. They will determine whether or not several axiomatic systems are consistent. They will know that an axiomatic system may have many Ainstances.@ They will know what it means for a set of axioms to be *independent* or *dependent*. They will have strategies for determining dependence or independence of axioms. They will know the difference between sentences and A(logical) statements@, which can have truth values attached, and they will know what *theorems* are. They will understand the role the *Law of Contradiction* and the *Law of the Excluded Middle* play in (the usual) logic. They will have considerable experience rephrasing complex statements in terms of the connectives *and* and *or, conditionals, biconditionals* and *negations*. They will have experience in determining the truth value of a complex statement from the truth value of its component parts. They will understand the difference between universal statements and existential statements. They will be able to form the negations of quantified statements and apply DeMorgan=s Laws to quantified statements involving the connectives *and* and *or*. They will be able to formulate the converse and contrapositive of conditional statements. They will know what Avacuously true@ means. They will recognize the terms *hypothesis* and conclusion. They will understand the difference between inductive and deductive reasoning. They will have some experience proving simple theorems using direct proof methods and using indirect proof methods (i.e. proof by contradiction). They will be introduced to the term *abstract* and the value of an abstract model as well as understand the role of simplifying assumptions in mathematical models. They will work with the axioms for a group and see many realizations of this axiom system including the integers under addition, the positive rational numbers under usual multiplication and modular arithmetic. They will know what a (binary) operation is. They may use technology (spreadsheets) to create Amultiplication tables@ for finite groups. They will have experience of formulating simple conjectures and proving them or and other results in group theory. Students will apply the method of proof by mathematical induction to obtain several formulas concerning the *infinite* set of positive integers.