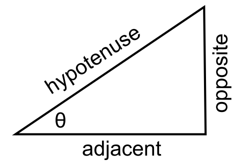
Useful equations



$\sin\theta = \frac{opposite}{hypotenuse}$ $\cos\theta = \frac{adjacent}{hypotenuse}$ $\tan\theta = \frac{opposite}{adjacent}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ Pythagorean Theorem: $c^2 = a^2 + b^2$ for a right triangle.

The quadratic equation is $ax^2 + bx + c = 0$, the solution to which is $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

$\vec{A}\cdot\vec{B} = AB\cos\alpha$, where α is the angle between $\vec{A}$ and $\vec{B}$.

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$.

$\vec{A}\times\vec{B} = AB\sin\phi$, with the direction determined by the right-hand-rule, and ϕ is the angle between $\vec{A}$ and $\vec{B}$.

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $v = \frac{2\pi r}{T}$ $a_{rad} = \frac{v^2}{r} = r\omega^2$

$f_s \leq \mu_s n$ $f_k = \mu_k n$

Missing variable	Translational Kinematics Equations	Connection between translation and rotational motions	Rotational Kinematics Equations	Missing variable
x	$v_f = v_i + a\cdot\Delta t$	$a_{tan} = r\alpha$	$\omega_f = \omega_i + \alpha\cdot\Delta t$	θ
a	$x_f - x_i = \left(\frac{v_i+v_f}{2}\right)\cdot\Delta t$	$v = r\omega$	$\theta_f - \theta_i = \left(\frac{\omega_i+\omega_f}{2}\right)\cdot\Delta t$	α
t	$v_f^2 = v_i^2 + 2\cdot a\cdot(x_f - x_i)$	$\vec{\tau} = \vec{r}\times\vec{F}$ $\tau = rF\sin\phi$	$\omega_f^2 = \omega_i^2 + 2\cdot\alpha\cdot(\theta_f - \theta_i)$	t
v	$x_f = x_i + v_i\cdot\Delta t + \frac{1}{2}\cdot a\cdot\Delta t^2$		$\theta_f = \theta_i + \omega_i\cdot\Delta t + \frac{1}{2}\cdot\alpha\cdot\Delta t^2$	ω_f

$E_{final} = E_{init} + W_{nc}$ $E = U_g + U_s + K_{translational} + K_{rotational}$

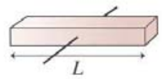
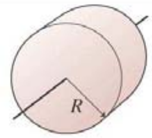
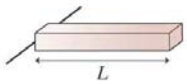
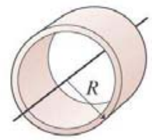
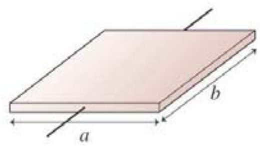
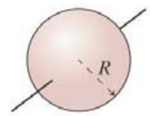
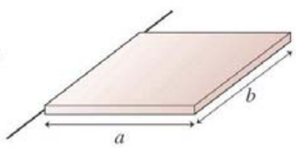
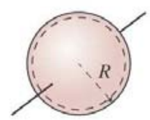
$K_{translational} = \frac{1}{2}mv^2$ $K_{rotational} = \frac{1}{2}I\omega^2$

$U_s = \frac{1}{2}kx^2$ $U_g = mgh$ (when on a planet)

$F_{gravity} = \frac{Gm_1m_2}{r^2}$ (when not on a planet)

The parallel axis theorem states that the moment of inertia about any axis that is parallel to and a distance d away from the axis that passes through the center of mass is given by $I = I_{cm} + Md^2$.

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

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$$I = \sum_i m_i r_i^2 = \int r^2 dm \quad \vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} \quad \vec{x}_{cm} = \frac{\sum_i m_i \vec{x}_i}{\sum_i m_i} \quad \vec{y}_{cm} = \frac{\sum_i m_i \vec{y}_i}{\sum_i m_i}$$

Translational Equations		Rotational Equations
$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$		$\omega_{average} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$
$v_{instantaneous} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$		$\omega_{instantaneous} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
$v = v_o + \int_{t_i}^{t_f} a dt$		$\omega = \omega_o + \int_{t_i}^{t_f} \alpha dt$
$a_{instantaneous} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt}$		$\alpha_{instantaneous} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d}{dt}\frac{d\theta}{dt}$
$x = x_o + \int_{t_i}^{t_f} v dt$		$\theta = \theta_o + \int_{t_i}^{t_f} \omega dt$
$\sum \vec{F} = \frac{d\vec{p}}{dt}$ $\sum \vec{F} = m\vec{a}$, for constant m		$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$ $\sum \vec{\tau} = I\vec{\alpha}$, for constant I
$\vec{p}_{tot} = \sum_i m_i \vec{v}_i$		$\vec{L} = \vec{r} \times \vec{p}$ $\vec{L} = I\vec{\omega}$
$\vec{p}_f = \vec{p}_i + \vec{J}_{ext}$		$\vec{L}_f = \vec{L}_i + \sum \vec{\tau}_{ext} \Delta t$
$\vec{J}_{ext} = \sum \vec{F} \Delta t$		
$\text{Work} = \int \vec{F} \cdot d\vec{x}$ $= \int F dx \cos\alpha$ $= FD\cos\alpha$		$\text{RotationalWork} = \int \vec{\tau} \cdot d\vec{\theta}$ $= \tau \Delta\theta$