

Uncertainty vs. Significant Figures (sig figs)

1. 2 values can have the same sig figs and different uncertainty

102 km +/- 1 km has three sig figs and an uncertainty of 1 km

1.02 km +/- 0.01 km also has three sig figs and an uncertainty of 0.01 km.

2. 2 values can have the same uncertainty and different sig figs

1.01 km +/- 0.01 km also has three sig figs and an uncertainty of 0.01 km

251.02 km +/- 0.01 km also has five sig figs and an uncertainty of 0.01 km.

3. RULES of sig figs:

Addition and Subtraction: Use the worst uncertainty

210.000 m + 10.0 m = 220.0 m because the worst uncertainty is 10.1 m +/- 0.1 m

210.000 m - 10.0 m = 200.0 m

(2100 m +/- 100 m) + 100.0 m = 2200 m +/- 100 m or 22×10^2 m

Multiplication and Division: Use the worst sig fig.

20.000 m x 10.0 m = 200. m

20.000 m x 10. m = 200 m +/- 10 m or 20×10^1 m

20.000 m x 10.000 m = 200.00 m

20.000 m ÷ 10. m = 2.0 m

20.000 m ÷ 10.0 m = 2.00 m

20.000 m ÷ 10.00000 m = 2.0000 m

20.000 m ÷ 1 m = 2×10^1 m

4. Whenever a quantity is measured, the absolute error should be estimated or determined from the equipment data sheet. For instance, $D = 3.42 \text{ cm} \pm 0.02 \text{ cm}$ or $f = 1025 \text{ Hz} \pm 5 \text{ Hz}$. Usually, there is uncertainty in the estimation. Just use common sense and justify your decision.

5. The absolute error in a calculated result is computed as follows. Determine (using estimations, reason, or your data) the MAX value; this is the largest the value can be, taking into account the errors in the quantities involved in the calculation. Next compute the MIN value; this is the smallest the value can be based on the estimated or determined errors. The absolute error is given by

Absolute Error = (Max Value - Min Value) / 2

For example, suppose speed = $(h_2 - h_1) / 100.0 \text{ s}$. Suppose further that $h_2 = 6.07 \text{ cm} \pm 0.02 \text{ cm}$ and $h_1 = 1.67 \text{ cm} \pm 0.04 \text{ cm}$ and 100.0 s is accurate to +/- 0.1 s. The MAX value is $[(6.07 \text{ cm} + 0.02 \text{ cm}) - [1.67 \text{ cm} - 0.04$

cm]) / [100.0 s - 0.1 s] = 0.0466 cm / s using correct significant figures. The MIN value is [(6.07 cm - 0.02 cm] - [1.67 cm + 0.04 cm]) / [100.0 s + 0.1 s] = 0.0434 cm / s

The absolute error is (0.0466 cm/s - 0.0434 cm/s) / 2 = 0.0006 cm/s

6. The BEST value is given by Best Value = (Max Value + Min Value) / 2

Using the example above the best value is (0.0466 cm/s + 0.0434 cm/s) / 2 = 0.0450 cm/s

7. Don't forget the rules of significant figures. When you add and subtract the result has the same number of digits as the number with the least number of digits (e.g. 1021 m - 0.0002 m = 1021 m). When you multiply and divide the result has the same significant figures as the number with the worst significant figure (e.g. 1021 m * 0.0002 m = 0.2 m²)

8. When calculating the MAX and MIN values don't forget the following rules

MAX via addition = (h1 + Δh1) + (h2 + Δh2)

MIN via addition = (h1 - Δh1) + (h2 - Δh2)

MAX via subtraction = (h1 + Δh1) - (h2 - Δh2)

MIN via subtraction = (h1 - Δh1) - (h2 + Δh2)

MAX via multiplication = (h1 + Δh1) * (h2 + Δh2)

MIN via multiplication = (h1 - Δh1) * (h2 - Δh2)

MAX via division = (h1 + Δh1) / (h2 - Δh2)

MIN via division = (h1 - Δh1) / (h2 + Δh2)

9. You can represent error in three ways

(a) Best Value +/- Absolute Error

(b) Best Value +/- Fractional Error

(c) Best Value +/- Percent Fractional Error

Where fractional error = absolute error / best value, and percent fractional error is 100% * absolute error / best value.

10. By convention the absolute error is expressed to only one significant figure. When you calculate the error round it up to one significant figure (never, ever round down your error. Doing this reduces your error and this is why bridges fall down). Once you have determined the one significant figure for your error round your best value to the same digit. For example, it makes no sense to say X = 2.48 cm +/- 0.5 cm. Rather, one should report X = 2.5 cm +/- 0.5 cm. Another example, it is reported t = 105 s +/- 0.4 s. Rather, one should report t = 105.0 s +/- 0.4 s.