Problem Set 11

1. Apply Hund's rules to find the ground state (in spectroscopic notation and the effective magnetic moment of the following species. Show all your steps.
   (a) Eu^{++}, in the configuration $4f^7 5s^2 p^6$.
   (b) Yb^{3+}
   (c) Tb^{3+}

2. The wave function of atomic hydrogen in is ground state (the 1s state) is

$$\psi(r) = \frac{1}{\sqrt{\pi a_o^3}}\, e^{-r/a_o}, \tag{1}$$

   where a_o is the Bohr radius. The charge density is given by $\rho = -e|\psi|^2$, according to the statistical interpretation of the wave function.
   (a) Show that for this state $\langle r^2 \rangle = 3a_o^2$.
   (b) Calculate the molar diamagnetic susceptibility of atomic hydrogen.
   Answer: -2.36×10^{-6} cm^3/mol (in cgs units), -2.98×10^{-11} m^3/mol (in SI units)

3. Suppose we have a crystal of N molecules each of which has an unpaired electron in a triplet state such that $j = l = 1$. Assuming no interactions between electrons of different molecules, prove that the low-field paramagnetic susceptibility is given by

$$\chi = \frac{2N\mu_B^2\mu_o}{3k_BT} \tag{2}$$

4. The most important contribution to the paramagnetism of $CuSO_4$ comes form the Cu^{2+} ions. These ions have a magnetic moment due to a single unpaired spin, where $l = 0$ and $j = s = \frac{1}{2}$. Suppose we have a solid of this material in a small magnetic field B.
   (a) What fraction of ions have a magnetic moment parallel to the field?
   (b) Show that the magnetization for N ions in the field B is given by

$$M = N\mu_B \tanh(\mu_B B/k_B T) \tag{3}$$

   (c) Find the internal energy of this system.
   (d) Find the magnetic specific heat, c_B. What are the limiting forms of this specific heat at high and low temperatures? Sketch c_B as a function of temperature.
   (e) What is a high temperature, if $B = 0.5$ T?