Problem Set 2

1. Find the Fermi energy of sodium (assuming 1 free electron per atom). State any sources you use (wikipedia is okay for this). What is the Fermi temperature of sodium?

2. If the Fermi energy for gold is 5.5 eV, what is the velocity of an electron at the Fermi surface (v_F)? Compare the Fermi velocity with the velocity derived assuming the electrons are a classical ideal gas.

3. The atomic bonding in a plane of graphite is very strong, but the bonds between planes are very weak. This system is of great current interest to the condensed matter community because it acts like a two-dimensional metal. Find the density of states and the Fermi energy for graphite, if N/A is the number of carriers per unit area. State your answers in terms of m, L, $\hbar$, N/A, and π. Estimate the Fermi energy in units of eV.

4. The bulk modulus at $T = 0$.

   (a) Show that the kinetic energy of a 3-D gas of N free electrons (when $T = 0$) is

   $$U = \frac{3}{5} N E_F, \tag{1}$$

   where E_F is the Fermi energy.

   (b) Find the relationship between pressure, volume, and the energy of the gas (still at $T = 0$). Recall from thermodynamics that $p = -\partial U/\partial V$. Compare this result with that of a classical ideal gas (which we all remember from PHYS-21700 as $pV = \frac{2}{3}U = Nk_BT$).

   (c) The bulk modulus is defined as $B = -V(\partial p/\partial V)$ (also from thermo). Show at $T = 0$ that $B = \frac{5}{3}p = \frac{10}{9}\frac{U}{V}$.

   (d) For potassium, estimate the value of the electronic contribution of the bulk modulus. State any sources. The book can be useful here.

   (e) *Extra credit* What percentage of the bulk modulus at $T = 0$ is due to the electrons in potassium? $T = 4.2$ K is a fine approximation for $T = 0$ for the purposes of this problem. Again, state any sources you used.