1. Consider a monatomic 1D lattice of N atoms with only nearest-neighbor interactions. From the dispersion relation for such a lattice, show that that density of states can be written as
$$g(\omega) = \frac{2N}{\pi} \frac{1}{\sqrt{\omega_m^2 - \omega^2}}$$
where ω_m is the maximum frequency allowed on the lattice.

2. Consider a 1-D chain of identical atoms of lattice spacing a, mass M, and force constant K between nearest neighbors. Using the Debye approximation for the dispersion relation with an assumed speed of sound v_s, find the Debye frequency. Develop an expression for the total energy due to phonons and find the specific heat at low temperatures.

3. Consider a dielectric crystal made up of layers of atoms with rigid coupling between layers so that the motion of atoms is restricted to the plane of the layer (i.e. a 2-D solid). Using the Debye approximation, obtain the expression for the thermal energy and determine the dependency of phonon heat capacity as a function of temperature in 2-D.

4. Estimate the number of phonons present in a 1 cm^3 piece of sodium at $T = 1\text{K}$. State all sources.