Teaching Calculus Students How to Study

Matthew R. Boelkins
Syracuse University
Department of Mathematics
Syracuse, NY 13244
mrboelki@syr.edu

Thomas J. Pfaff
Syracuse University
Department of Mathematics
Syracuse, NY 13244
tjpfaff@syr.edu

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Abstract

The authors address the problem of poor study habits of calculus students and present techniques to teach students to study consistently and effectively. The approach explicitly shows students how to spend their time studying at least an hour a day, six days a week. It also helps them improve both their learning of new material and retention of earlier concepts in a highly organized fashion. Many students greatly appreciate the added structure, work harder than in previous courses, and witness newfound success as a consequence.

KEYWORDS: calculus, study skills, syllabi, review

INTRODUCTION

Most instructors agree that college students should spend at least two hours studying outside class for each hour in class. Indeed, this standard is a reasonable one, for such a time investment is necessary for the average student to be successful, particularly in mathematics courses. Unfortunately, we have found that few undergraduates spend this much time studying for calculus courses and many seem to lack an understanding of how to structure and organize their efforts.

Throughout the ranks of college teachers, there is dismay over the apparent lack of work ethic among students. In his recent book Generation X Goes to College [2], Peter Sacks laments the state of today's college student. He claims that the typical postmodern student is essentially a consumer seeking grades in return for tuition dollars and has little desire to expend a serious effort to learn. Other authors like Steven Zucker [3] and David Olson [1] have also noted the absence of work ethic and grade-based motivation of students in the 90's, and have written of their efforts to change this behavior in the mathematics classroom. We feel that this missing work ethic is not due to an inherent laziness in students, nor to an inability to become diligent. Many of our students have jobs or participate in extracurricular activities at which they work extremely

hard. Rather, we believe the apparent lack of willingness to work at studying is due to an absence of explicit knowledge of what is necessary to succeed in the academic arena.

In fact, we have found many students who both want to give a serious effort and find satisfaction in solving the difficult problems calculus presents. In the words of one of our students from a recent course evaluation,

> I like this class because it is challenging.

(Subsequent indented quotes in this paper are verbatim remarks from student evaluations.) We recently discovered a way to give our students concrete ideas on how to study mathematics, and they have greatly improved their work habits and enjoyed newfound success as a consequence.

AN EYE-OPENING SEMESTER

In a recent semester we were teaching, respectively, the third and fourth courses in the calculus sequence. As graduate teaching assistants, it was a new experience for each of us to teach a class that was not predominantly first year students, and we were excited to work with more mature, experienced college students. Each of us was (naïvely) optimistic that we were finally teaching classes that would not need constant prodding.

In one of our courses, the first exam was a complete disaster. It was clear that the students were not even close to understanding the material and some serious problems were present. Casual questioning uncovered the fact that many students were coming to exams largely unprepared. Further investigation in both of our classes revealed that a high percentage of students were spending only an hour or two a week studying, and that exam preparation consisted essentially of a few hours the night before or morning of the test. One student spoke for many of his peers when he wrote,

> In a typical non-test week, I didn't study at all.

We concluded that our students lacked an understanding of study techniques, despite having completed a year or more of college. They didn't have a comprehensive strategy for learning and retaining the material, nor did they recognize the importance of working consistently and regularly. The students also seemed completely unaware of relatively standard techniques and ideas for studying mathematics. For instance, almost no one used a brief outline or 3×5 index cards to help them remember definitions, important ideas, theorems, key examples, and the like. Moreover, students seemed to rarely review problems they had already done, try new problems in the book that weren not assigned, or choose problems from a variety of sections to test their understanding. While we implicitly expected our students to do these things, we did not effectively communicate this need to them. We knew our students could not be successful unless their study habits changed.

We set out to reform the way in which our students were approaching their study of calculus. Our goal was to show students how to develop a general,

consistent plan for studying throughout the semester. It was assumed that if students indeed began to work regularly, success would be nearly unavoidable, giving them more encouragement to continue their new study habits.

OUR FORMER METHOD

In our department, the usual regimen for a calculus class is as follows. All the instructors of a given course receive a standard, department syllabus from the "course chair" at the start of the semester. This department syllabus also contains a semester schedule with the section(s) for each class meeting, along with a corresponding selection of problems. Our classes are three credit hours, and usually meet on a MWF schedule, so there are assigned problems for three days each week. An exerpt from a typical week on a department syllabus follows.

Mon.	Sec 13.1	# 1,7,15,17-22,29,31,33,35
Wed.	Sec 13.2	# 2,8,13,15,28,31,35,37,41,47,53
Fri.	Sec 13.3	# 3,5,9,13,15,17,21,22,23,27,33,35

Until recently, we used this schedule in the syllabus as our central means to inform students of the necessary work.

This method of communicating the minimum required work has several weaknesses. First, it appears that this calendar led students to conclude that they should work on calculus at most two or three days per week. Second, it seemed to force them to try both the routine and the challenging problems all at once, and this led to much frustration. Third, because the textbook groups problems essentially by type, and the course syllabus does likewise, even students who were following the syllabus ended up doing collections of problems which were usually quite similar to one another. This created difficulties in learning to apply important ideas in varied contexts. And finally, despite our pleas to the contrary, many students perceived that any problems in these pages, other than those specifically collected for a grade, were simply optional and only for people with nothing better to do.

A NEW TACK

Our new approach, while simple, had a profound effect. Each Monday we gave our students a "problem sheet" for the coming week. This schedule specifically listed the work to be done for six days of the coming week. A typical day on this schedule included routine homework problems from the most recent lecture, harder problems from the last couple of classes, and a few review problems. Also included were specific study directions for students to create note cards, outline material, or review previous homework. Through this use of a daily schedule, we broke the material into smaller bites and spread new ideas out over more time. In addition, the very act of giving a new set of assignments each week, rather than presenting the entire semester's schedule at once, regularly called students' attention to the work that needed to be done. To see the difference students saw in this presentation as compared to a typical week in the department syllabus above, an exerpt follows. This example is from the same week as the previous one.

Mon.	Sec 11.6	45,49,65,71	outline ch 12
	Sec 13.1	1,7,15,19	
Tue.	Sec 13.1	29,31,33,35	notecards for 13.1
	Ch 12 rev	7,10,19	
Wed.	Sec 13.1	22,39,47	review for Fri quiz: ch 12, 13.1
	Sec 13.2	2,8,13,15	
Thu.	Sec 13.2	28,31,37,41	review for Fri quiz
	Ch 12 rev	21,23,27,39	
Fri.	Sec 13.2	47,53	notecards for 13.2
	Sec 13.3	3,5,9,13,15,17	
Sun.	Sec 13.3	21,23,27,33,35	notecards for 13.3
	Sec 11.5	22,25,27	

This format gives students clear direction to work with easier problems first to allow time for questions and some study before progressing to harder ones. It also offers other problems from a variety of sections to aid retention and help students prepare for exams. Furthermore, this method presents an ideal forum in which to include specific study suggestions. We were extremely explicit, always stating reasonable, carefully chosen daily tasks. It quickly became evident that our students were benefitting from (and appreciating) the added structure.

> The problem sheets were a great gift; they organized me.

Along with assigning daily work to our students, we entered an ongoing dialogue with them about what they needed to do to be successful. For example, we showed how to create an outline or notecards to study from, and explained the importance of doing this well in advance of exams, in order to review the ideas several times. We also pointed out that such notes are of great use for the final exam, and even future courses. Such discussions were rarely long, usually 5-10 minutes at the start of class once or twice a week, but the constant focus on studying, together with giving them a detailed schedule each week, gave students the element of structure they were previously lacking. It was not difficult to convince our students that it was much easier to study an hour or so a day than to try to do eight hours of calculus in a single sitting, especially when we offered them alternate strategies.

Finally, we must note that the rapport we established with our students was critical to our success. While creating the daily schedule for each week does not require this, our ongoing discussions of studying were greatly enhanced by knowing our students well. Their trust in us helped them make a willing transition to a new approach to studying that involved a substantial commitment.

> He honestly seems to care if his students understand and learn what he's teaching. I've found this to be a rare trait amongst instructors in my four years of college.

This comment again demonstrates the problem. We consider it unlikely that most of this student's instructors didn't care if she understood or not – rather, it was probably the case that they did not *show* that they cared. Much of our effort centered on making our implicit concerns and expectations explicit.

CHANGED BEHAVIOR

Both of us saw immediate results once we implemented the structure of the weekly problem sheets, together with our discussions on studying. In the fall calculus III class, this method was begun immediately following that disastrous first test. The other author instituted this approach the first day of the spring semester when teaching calculus IV for a second time. What we saw in our students was both enlightening and exciting.

The sight of students with notecards and brief outlines covering the main ideas became the norm, rather than the exception. As we interacted with students in office hours, it was common to see notebooks full of completed problems and to be asked very specific questions reflecting much thought. In addition, it became typical that students in our classes had questions each day which revealed an already substantial effort on their part. This made the environment more conducive to teaching new material. Our students were benefitting from having a solid foundation of understanding on which to build new ideas.

Watching the vast majority of our students doing so much work was perhaps the most pleasing result. Several students could be found doing the new homework assignments in the nearby study room immediately after class ended each day. Given our past experiences, we were at first astounded as we saw student after student with pages of neatly worked problems, and asterisks next to the few that were confusing. Previously, students came to office hours and said "I can't even get the first problem," and that was the first problem from two weeks ago! Now students had nearly all the problems done and it became common to hear them say things like, "I'm comfortable calculating these directional derivatives, but can we talk some more about what they really mean?"

Not only were students now doing a tremendous amount of work, but they were doing it on schedule. One of the authors had a student come to his office one day and begin with the words, "I'm really sorry. I'm behind, but I'm trying to catch up. Could you help me with some of the older stuff?" This occurred on a Friday morning before class, and when the student pulled out his book and assignment list, he pointed out several early problems from Monday (of that very same week!) that he was struggling with. This was in stark contrast to behavior we had seen in previous semesters where students two and three *weeks* behind seemed to think nothing of it.

AMAZING RESULTS

Given all of the effort our students were now expending, it is not surprising that we also saw a tremendous increase in the quality of work we were reading. Because most students completed all of the problems, the few that were collected for a homework grade were generally correct and well thought through. In past semesters, the collected homework questions were often the *only* problems the students did, and it was not uncommon to have blank problems or entirely erroneous work turned in. More importantly, the general performance on exams went way up as well. In the calculus III class where these changes

were implemented after the disastrous first test, there was a 25 point rise in the class average on the second test.

> This new style of taking smaller chunks more often, and reviewing frequently was very helpful. I think this made everyone do better.

It might be tempting to think that we simply began giving easier tests in an effort to reward our students. On the contrary, we actually found that we were able to give exams that were more challenging and greater in length. Because we knew our students were working hard, we grew less shy about asking challenging questions.

Another anecdote exemplifies the great changes we witnessed. On a Friday, prior to the Monday of the first calculus IV exam of the spring semester, one of the authors informally asked his class how many people had already studied at least 3-4 hours specifically for the test (in addition to keeping up with the new material). At least 75% of the class raised their hands.

> The daily assignments kept me doing the work on a consistent basis.

Indeed, the survey in the above class showed that although it was still three full days before the test, more studying for the exam had been done by the average student than was often the total effort by a person in previous classes. Even test taking behavior changed. It became common to have most of our students stay for the entire exam period, working until we demanded they turn in their exams. Most students were able to quickly complete the more straightforward questions, and then used all of the remaining time to tackle the problems requiring more novel thought. Overall, our classes had a better average performance than previous ones who took tests we deemed markedly easier.

FINAL EXAMS

The strongest evidence that our students had improved came from their performance on final exams. At our institution, mathematics students in a particular course take a common final exam. It is written by the course chair, often with the input of the other instructors, and is graded collectively by all the instructors of the various sections. In the fall calculus III class where our new approach was first implemented, and mid semester at that, the class performed above the overall average on the final, with several truly outstanding results. In the spring calculus IV class where the weekly problem sheets were used throughout the semester, the two sections of one of the authors far outperformed the other three sections of the course.

To see the actual breakdown of scores, let "A" represent the combined 62 students in the three sections taught by other instructors, and "B" stand for the 55 students in the two sections of one of the authors. For A, the median score on the 150 point final was 97/150. In B, where the focus on consistent studying was employed, the median was 115/150. The distribution of scores is even more revealing. In what follows, the category "130" indicates the number of scores betweeen 130 and 139, and so on.

	140	130	120	110	100	90	80	below 80
A	0	2	5	6	14	10	8	17
B	2	6	16	7	10	7	4	3

This breakdown of results shows that not only did students who had been specifically encouraged to study daily achieve the very best scores on the final, but also almost all of these students passed the exam with a score of 90/150 or better. In fact, only 7 of the 55 students in group B failed the final, while 25 of the 62 in group A did. Results like that seen in group A, unfortunately, are not uncommon in the calculus sequence, and we ourselves saw such performances prior to instituting our emphasis on study techniques.

STUDENT AFFIRMATION

The level of performance that we saw our classes achieve speaks volumes about the effects our change in approach had on them. Moreover, students' own comments demonstrate this. The following quotes come from teaching evaluations and end-of-semester surveys from the classes in which we employed the techniques noted above. These comments show that students were willing to work and most appreciated the structure of the course. The first comment especially evidences the change the students noted in the class where these ideas were implemented after the first exam.

> The instructor took a great deal of responsibility for how we did in the class. After the first test when people hadn't done well, he adjusted his teaching style and grades improved. He took more time to answer questions at the beginning of class, he gave us adapted, nightly homework, and had us change our study habits. All of this improved the grades on the second exam immensely. I felt that the instructor has an excellent attitude, and his ability to adapt his teaching style was helpful.

> Class structure was very helpful. The instructor gave a plan on how to study effectively and review daily.

> The problem sheets were most helpful. I did all of the problems and kept them organized so that I could study from them.

> The problem sheets were the best way to study. It kept your mind on calculus regularly, and since I worked at generally the same time period each day, it became a mental routine.

SOME CHALLENGES

One side effect of getting students to study more was that there was a mild increase in complaining about grades. Since our students were working far harder than they ever had before, they naturally expected higher grades. Many,

of course, did see improvement over their past performances, but in an era when B's are perceived as average, it was difficult to explain to students that a C+ was an acceptable performance on a challenging exam. In addition, this problem was compounded by the fact that many of our students realized that they were working significantly more than their peers in other sections of the same course. We remain unsure of how to address these complaints about letter grades, other than to continue to emphasize that the importance of learning the material far surpasses that of grades.

It is also worth addressing a concern of some of our colleagues, that being that second and third year college students should not need to be taught how to study. We agree, but having noted the general college student's lack of study skills, we feel we were (and are) faced with no alternative. What can be done to change this is to begin focusing such efforts and training in first year courses. We've seen that sophomores and juniors are very willing to learn and use such skills, and we expect that first year students would be at least as responsive. The consistent study skills we taught are ones that students learn quickly, and will repeat because they enjoy the success that comes with them. Even within the semester time period of our classes, we found that we had to do less and less explicit explaining of studying. If in fact students learn these techniques during their first year, it would no longer be necessary to spend substantial time on these ideas with sophomores and juniors.

Much of our response to this problem of students not studying came from listening to our students' comments. It is therefore important to address the complaints we did receive in our evaluations which were related to our new strategies. We present these here with brief responses:

> Sometimes the assignments seemed very long and overbearing. Often that led me to try too long to catch back up, which brought me even further behind. A couple of the tests were very long, and the length of the class time seemed unrealistic for many people.

At times the assignments may indeed have been too long. It took time to get a feel for how much the students could accomplish in an hour or so a day, and at first we may have been overly ambitious. This student talks about trying to catch up. Indeed, falling behind in our regimented study schedule is deadly. We required a substantial daily effort, so we can see why he may have thought the assignments were too much. This student also refers to the length of class time for exams. Our tests were challenging, and while most students remained for the entire hour, almost everyone was able to complete the exams. Part of what we continued to try to convey to students was that a complete understanding of the material enabled one to work quickly, and we expected this on routine questions.

> I felt that too many homework problems were given and at times working through them seemed really monotonous. The only other thing I really didn't like was how so many problems/collected prob-

lems were due on the same day as the half hour exams. It made the work doubled (studying and doing problems at the same time).

It is possible that, for our very best students, the assignments became monotonous. We did ask them to do a lot of problems, but only a handful of them were collected for a grade. It was left to the students' judgment as to whether or not they needed to do every problem, though they were strongly encouraged to do so. This student also references problems due the same day as half-hour exams. He seems to have missed the point that *doing problems IS studying!* This idea is one we will continue to communicate to our classes.

FINAL THOUGHTS

When we hear students say things like "in a typical week, I didn't study at all" it is easy to become cynical and complain about the general malaise. But the fact is, such cynicism is good for no one: students perceive it and get turned off, and teachers become less and less interested in teaching. We have seen that today's students are indeed capable of learning and working hard. Our students learned that consistent, effective studying, much like the daily practice of musicians and athletes, can produce amazing results. Teaching study skills addresses the problem of getting students to work on their own and has the desired effect of helping them learn the material.

In [3], Zucker discusses his efforts to bring his students up to the level he expects, mainly by being explicit about the fact that much learning at the university level must occur on one's own. He has directed his efforts at his students, more than the material itself, stating his belief that "We should be ... reforming the *students*, not the calculus!" [3, p. 864] While there is room for calculus reform, what should not be overlooked is the ability we have to reform and reshape our students. The success we have found in teaching our students how to study compels us to continue these efforts in future classes, where we expect to see more students learn how to work hard and meet the challenges of calculus.

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BIOGRAPHICAL SKETCHES

Matt Boelkins is a doctoral candidate and graduate teaching associate in the department of mathematics at Syracuse University. He wants to teach college mathematics for a living and continue to find ways to excite students with the beauty and importance of the subject. When not teaching calculus or trying to finish his dissertation on the spectral radius of positive operators, he enjoys the occasional round of golf, as well camping and hiking with his wife and son.

Tom Pfaff is a graduate teaching associate at Syracuse University where he is working on his Ph.D. in probability along with a second M.S. in applied statistics. After receiving a B.S. in exercise science from Ithaca College, he went to SUNY Cortland for two years to finish a B.S. in mathematics before entering graduate school at Syracuse. He hopes to return to the college environment, this time as a faculty member, and introduce undergraduates to the challenges of mathematics as well as its wealth of "recreational" problems.