

## Special Session on RUME Abstracts

This special session, co-organized by Sarah Hansuch (SUNY Oswego) and Aaron Weinberg (Ithaca College), is co-sponsored by the MAA SIGMAA-RUME. Presentations include rigorous and scientific studies about students' mathematical cognition and reasoning, studies of teaching practice in inquiry-oriented mathematics classrooms, and design of research-based curricular materials. Presentations build on the existing literature in mathematics education and use established or innovative methodologies as they pertain to the study of undergraduate mathematics education.

**Ellie Fitts Fulmer, Ithaca College**

*Designing Intellectual need-Provoking Tasks for Introductory Calculus*

Abstract: Students in undergraduate mathematics classes are routinely asked to learn from textbooks. In recent years, mathematics and literacy researchers have begun to investigate the ways students learn from discipline-specific texts using the perspective of disciplinary literacy, which focuses on how experts interpret, create, and critique disciplinary texts such as mathematics journal articles. However, textbooks differ from other disciplinary texts because they are specifically prepared for classroom use. Our work analyzes the reading practices of undergraduate calculus students and non-mathematics STEM professors as they interact with excerpts from calculus textbooks. We have proposed the idea of didactical disciplinary literacy to describe the productive reading practices we observed, and this paper zeros in on the role of readers' agency.

**Sarah Hanusch, SUNY Oswego**

*Feedback on Proofs: An analysis of faculty practices*

Abstract: Mathematics faculty spend considerable time scoring and providing feedback on student-generated proofs, yet this practice is largely unresearched. In this talk, I explore the types of annotations that professors make on student proof attempts, and the manner in which the feedback is phrased. The results show that professors generously use annotations (like checkmarks) as informal grading tools or to signify things they have read when grading, most feedback focuses on a particular part of the proof that is no more than a few lines, and the majority of feedback does not convey why the feedback was given.

**Jessica Tornai, Ithaca College**

*Attentive Fidelity: What do Students Pay Attention to in Calculus (Video) Lectures?*

Abstract: When students watch an instructional video or a lecture, do they pay attention to what the instructor thinks are the most important features? What aspects do students focus on and how does this impact their learning? We report on research using eye-tracking methodology with instructional calculus videos to investigate students' attentive fidelity—the degree to which they attend to the visual imagery that is the subject of the video narration at each moment in time. We describe what students attend to and whether this is correlated to their learning from watching the video.

**Aaron Weinburg, Ithaca College***How do College Students Read Calculus Textbooks? Using a New Theory to Understand Agency in Didactical Disciplinary Literacy*

Abstract: Intellectual need is the need that students feel to understand how and why a particular mathematical idea came to be. We are interested in creating tasks that calculus instructors can use to provoke intellectual need. However, the current suggestions for designing such tasks lack detail and don't account for several issues specific to undergraduate introductory calculus. In this theoretical paper, we discuss the idea of intellectual need, explore three issues related to the teaching of calculus, and present a theoretical model that task-designers can use to frame important factors that affect the development and use of these tasks.

Contributed Talk Abstracts**Anurag Agarwal, Rochester Institute of Technology***Solutions for some quadratic Diophantine equations*

Abstract: We will discuss and investigate the positive integer solutions of some quadratic equations whose solutions have links to generalized Fibonacci and Lucas sequences.

**Cesar Aguilar, SUNY Geneseo***Eigenvalues of threshold graphs*

Abstract: Problems in algebraic graph theory provide a rich source of research projects for undergraduate students. In this talk, I will present some results obtained over the last couple of summers with SUNY Geneseo undergraduates on the study of the eigenvalues of threshold graphs. The main takeaway of the research is that there is a distinguished threshold graph that plays a prominent role in the study of the spectral properties of the entire class of threshold graphs.

**Abd AlRahman AlMomani, Clarkson University***Directed Partitioning : Theory, Applications, and Challenges*

Abstract: In this work, we discuss the graph directed partitioning method, and its applications in complex systems science such as but not limited to coherent structures, computer vision, weather, complex networks analysis, and earth science. We introduce examples and applications from Jupiter, weather movies, network synchronization, and predicting ice shelf cracks in Antarctica's Larsen C ice shelf. *Authors: Abd AlRahman AlMomani and Erik Bollt (Clarkson University).*

**Ahmad Almomani, SUNY Geneseo***Locally Anchored Swarm Optimization (LASO)*

Abstract: In the recent decade, Particle Swarm Optimization (PSO) become a favorable global optimization method the fields of science and engineering. Moreover, PSO is a metaheuristic method, and it makes few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions, which made it an efficient method in the field of machine learning, and training of neural networks. However, two main problems face the PSO which are the possibility to trap with local minima and the slow local convergence. This work introduces an efficient method to combine the Swarm Optimization with the Local optimization solvers, which goes beyond the parallel independent implementation to use dynamic internal connections that achieve robust results.

*Authors: Abd Alrahman AlMomani (Clardson University) and Ahmad Almomani (SUNY Geneseo).*

**Doug Baldwin, SUNY Geneseo***Making an OER Calculus Text Our Own*

Abstract: Since academic year 2017-18, SUNY Geneseo's mathematics department has allowed instructors to use an open educational resource (OER) textbook (Openstax Calculus Volume 1) on a trial basis in its first calculus course. Many of our instructors are enthusiastic about this text, except for the large number of typographical errors it contains. During the summer of 2019, we took advantage of the book being an open resource to correct those errors. The result is a custom version of the book that is currently being used as the main text in 7 out of 9 sections of Calculus 1, and as an optional text in another section. In this talk we describe how we carried out this project, the results we have observed so far, future plans for the book, and lessons learned. We hope the project can serve as a model for others interested in using or adapting OER mathematics texts. *Authors: Christopher Leary, George Reuter, Gary Towsley (SUNY Geneseo).*

**Matt Coppenbarger, Rochester Institute of Technology***Iterations of the Sisyphus Function*

Abstract: The Sisyphus function is defined and we determine the smallest nonnegative integer  $n$  requiring a specified number of iterations of the function that must be applied to  $n$  until the sequence generated by the iterations of this function becomes stable or cycles.

**Matt Hoffman (Rochester Institute of Technology), Nicole Juersivich (Nazareth College), and Carl Lutzer (Rochester Institute of Technology)**

*Data Integration in Undergraduate Mathematics Education (DIUME)*

Abstract: We will describe our efforts in creating and evaluating the impact of teaching modules based on real-world data so that students have authentic experiences that support and motivate the investigation of concepts and techniques in calculus and linear algebra. Specifically, we looked at (1) how student disposition toward real-world data and the use of technology as a mathematical tool evolved in a course that used the modules and (2) how the completion of the data-driven and technology-integrated modules impacted student achievement in the course. We have collected data from pre and post-module student surveys, pre and post-module student focus groups, student final exam scores, instructor journals, and instructor interviews from multiple courses across our two institutions. We are now in our third year and would like to invite other institutions into the project. During this workshop, we will share our findings to date, takeaways we have learned throughout the study, and the digital modules and supporting technology files. A few computers with MATLAB will be available during the session along with hard copies of the modules so that you can explore and ask questions. To preview the module files, go to [shorturl.at/csvCS](http://shorturl.at/csvCS).

**John Maceli, Ithaca College**

*Mathematical Card Tricks*

Abstract: This talk will introduce some mathematical card tricks and their uses in the classroom. Many magic tricks are based on mathematics. We will discuss a few card tricks and the mathematics behind them.

**James Marengo, Rochester Institute of Technology**

*An Upper Bound for a Cyclic Sum of Probabilities*

Abstract: Let  $x_1$ ,  $x_2$ , and  $x_3$  be real numbers and consider the three statements  $x_1 > x_2$ ,  $x_2 > x_3$ , and  $x_3 > x_1$ . Clearly, these statements cannot all be true. But suppose that  $x_1$ ,  $x_2$ , and  $x_3$  are realizations of random variables  $X_1$ ,  $X_2$ , and  $X_3$ , and that the corresponding statements are each true with the same probability  $p$ . Since  $p$  cannot be equal to one, the following question arises: how close to one can  $p$  be? Can  $p$  be greater than  $1/2$ ? Can  $p = 0.7$ ? One can ask a similar question for  $n$  random variables. After mentioning a preliminary result that answers these questions and which will be proved in a student talk at this conference, we will show that, given a univariate absolutely continuous probability distribution an  $n - \epsilon > 0$ , there are random variables  $X_1, X_2, \dots, X_n$  having this distribution and for which each of the probabilities  $\Pr(X_1 > X_2)$ ,  $\Pr(X_2 > X_3)$ ,  $\dots$ ,  $\Pr(X_{n-1} > X_n)$ , and  $\Pr(X_n > X_1)$  exceeds  $1 - \epsilon$ .

**Sedar Ngoma, SUNY Geneseo**

*On a time-dependent inverse source problem with an integral constraint*

Abstract: We investigate an inverse time-dependent source problem for a parabolic partial differential equation with a Neumann boundary condition and subject to an integral constraint. We show the existence, uniqueness, and continuous dependence of solutions. The proof of the existence and uniqueness of solutions yields an algorithm that we used to approximate solutions of the inverse problem using a finite element discretization in space and the backward Euler scheme in time. The errors resulting from our experiments show that the proposed scheme approximates solutions of this inverse problem accurately.

**Sam Northshield, SUNY Plattsburgh***Tropical Cycles*

Abstract: The equation  $f(n+1)f(n-1) = f(n) + c$  was introduced and studied by Lyness in 1947. When  $c = 0$  or  $c = 1$ , and only then, the equation is “globally periodic” – i.e., every solution is periodic with the same period.

Tropical mathematics is had by replacing  $x$  by  $+$  and  $+$  by  $\min$ . The tropical Lyness equation is then  $f(n+1) + f(n-1) = \min(f(n), c)$ . It turns out that every solution is periodic (with some period) but the equation is globally periodic only if  $c = 0$  or  $c$  equals infinity.

We study these two cases as well as the other “tropical cycles”, i.e., globally periodic equations of the form  $f(n+1) + f(n-1) = F(f(n))$ , where  $F(x)$  is one of  $x$ ,  $0$ ,  $-x$ ,  $\min(x, 0)$ ,  $\max(x, 0)$ ,  $\min(-x, 0)$ ,  $\max(-x, 0)$ ,  $|x|$ , or  $-|x|$ .

**Gabriel Prajitura, The College at Brockport***Orthogonality without inner products*

Abstract: We will discuss and compare various concepts of orthogonality of interest in spaces without inner products and will look into their particular forms for the  $p$  norms (with  $p$  different from 2) in two dimensions.

**Alex Rennet, University of Toronto, Mississauga***A Report on multiple Large-Class Active Learning Redesigns*

Abstract: In this talk, I will outline the structure of Active Learning redesigns of large (500+ student) Calculus and Linear Algebra courses at the University of Toronto, Mississauga. We focused on creating a number of in-class and out-of-class components with the intention of maximizing student engagement during class, including online quizzes, polling questions, readings, and in-class activities. I will report on successes, challenges, and next steps for the redesigns. (These redesigns are still in the process of being implemented and adjusted, so this is an interim report. Each project was in collaboration with other faculty.)

**Paul Seeburger, Monroe Community College***Using the LibreTexts Platform to Customize OER Textbooks for Calculus II and III*

Abstract: The presenter will share his experiences using the LibreTexts platform to customize OpenStax textbooks for his Calculus II and III courses. LibreTexts includes a WYSIWYG content editor to seamlessly edit the textbook content, using LaTeX only where needed to format math content. You can add your own sections, subsections, examples, and exercises using a consistent numbering system to form a textbook that looks professional and is customized for your course. Using CalcPlot3D, rotatable 3D figures can be added to bring the figures in the textbook to life. Anyone can use these textbooks on the LibreTexts platform or customize them for their own courses. See [https://math.libretexts.org/Courses/Monroe\\_Community\\_College](https://math.libretexts.org/Courses/Monroe_Community_College).

## Student Session Abstracts

**Andrew Ditzel, SUNY Oneonta**

*The Congruence of Curves in the Three Dimensional Space*

Abstract: In this presentation, we will discuss the notion of congruence for curves in the three dimensional space. In particular, we will see that a necessary and sufficient condition for two curves to be congruent is that they have the same curvature and torsion. Some authors claim that this theorem represents in fact an analogue for curves of the criteria of the congruence of triangles from the two dimensional plane. In order to understand these concepts, we will start by discussing isometries and we will follow the so-called Frenet approach to differentiable curves. Among other things, we will see how the basic Frenet vector fields look like and how to express their derivatives in terms of the vector fields themselves.

**Matthew Ficarra, SUNY Geneseo**

*Tridiagonal Matrices with Continued Fractions*

Abstract: In this talk, we derive an alternate form to the recurrence relation of the determinant of a tridiagonal matrix using continued fractions. We then apply our derivation to obtain properties of the eigenvalues of a general threshold graph including the alternating behavior of the magnitudes of the eigenvalues about the value  $-1/2$  as well as obtaining equations whose intersections yield the eigenvalues of any threshold graph.

**Ryan Gelnett, SUNY Oswego**

*Folding Polyominoes*

Abstract: Continuing the work of Greg Fredrickson, Julia Martin, and Elizabeth Wilcox, for my summer research project I dove into studying folding polyominoes from one-level to two-levels. I classified a few infinite sets of polyominoes that are and are not foldable when restricted to two “legal moves” and along the way I also determined an algorithm to efficiently create foldable polyominoes from non-foldable ones.

**Emily Hampston, The College at Brockport**

*Prime numbers in between Fibonacci numbers*

Abstract: I will discuss the existence of prime numbers in between consecutive Fibonacci numbers and in between terms of other linear recursive sequences. *This work was conducted with Justin Kipp.*

**Megan Hardenbrook, The College at Brockport**

*An Alternate Method of Finding Maximum and Minimum of a Multivariable Function*

Abstract: I will show how the extrema of a multivariable function can be found using one variable techniques.

**Laynie Jensen, SUNY Cortland***Modeling Slime Mold Decision-making: The U-shaped Trap Problem*

Abstract: In biological systems, decision-making is an integral factor in organismal behavior, yet we still do not understand the processes behind it. Modeling the behavior of simple organisms helps us to understand the mechanisms and reasoning that directly result in the behavior of an organism. Single-celled slime mold *Physarum polycephalum* is capable of making complex decisions, all while lacking a nervous system or any nerve-like structures. What is unique about *P. polycephalum* is that it has external memory in the form of the secretion of a repellent chemical trail, which deters the slime mold from returning to previously explored areas. The attractive Keller-Segel model is a well-known model for predicting how slime mold *Dictyostelium* moves. Preliminary numerical analysis of the one-dimensional repulsive Keller-Segel model using a pseudospectral method confirm the results of our stability analysis on the model, and suggest that it can be applied to the movement of *P. polycephalum* as it navigates a U-shaped trap.

**Justin Kipp, The College at Brockport***Prime numbers in between Fibonacci numbers*

Abstract: I will discuss the existence of prime numbers in between consecutive Fibonacci numbers and in between terms of other linear recursive sequences. *This work was conducted with Emma (Emily) Hampston.*

**Quinn Kolt, Rochester Institute of Technology***An Upper Bound for the Sum of Cyclic Probabilities*

Abstract: Let  $x_1, x_2$ , and  $x_3$  be real numbers and consider the three statements

$$x_1 > x_2, x_2 > x_3, \text{ and } x_3 > x_1. \quad (1)$$

Clearly, these statements cannot all be true because if that were the case, it would follow, for example, that  $x_1 > x_1$ , which is a contradiction.

But, suppose that  $x_1, x_2$ , and  $x_3$  are realizations of random variables  $X_1, X_2$ , and  $X_3$  respectively and that each of the statements corresponding to those in (1) is true with the same probability  $p$ . That is,

$$Pr(X_1 > X_2) = Pr(X_2 > X_3) = Pr(X_3 > X_1) = p.$$

Since  $p$  cannot be equal to one, the following question arises: how close to one can  $p$  be? Can  $p$  be greater than  $\frac{1}{2}$ ? Can  $p = 0.7$ ? One can ask an analogous question for  $n$  random variables.

To answer these questions, we derive an upper bound for a cyclic sum of  $n$  probabilities, each of which involves inequalities for  $L$  random variables that are consecutively-indexed mod  $n$ , where  $L \in \{2, \dots, n\}$ .

**Alexandra Lewis, SUNY Oneonta***Coding with Application*

Abstract: Using RStudio application, we developed an R Dashboard Shiny App for ranking 4-year colleges and universities in the US in terms of their 4-year graduation rates. We created functions that read an external data file and returned the name of a 4-year college or university that has the “best” or the “worst” 4-year graduation rate in a particular state. Other graduation outcomes was also be considered. In addition, the App has the ability to take in arguments, such as the name of a state, and a ranking value of a 4-year college or university in that state. Then, the App will return the name of the college that has the specified rankings requested. Moreover, the App can also be used to display a leaflet map and information about the names of the 4-year colleges or universities that are the “best” or “worse” in their respective states based on their 4-year graduation rates and other outcomes. *Authors: Alexandra Lewis, Ryan Minges, and Christopher Robertson (SUNY Oneonta).*

**Una MacDonald, The College at Brockport***Probabilities in Number Theory*

Abstract: I will discuss what is the probability that certain sums end up with the same digits.

**Molly Marshall, SUNY Geneseo***Standing in a Room Full of Mirrors*

Abstract: Imagine yourself standing in a room full of mirrors, each direction you look there are surrounding copies of you, following each movement. This is what it is like to stand in a platycosm. There exist only 10 varieties of this effect, and in this presentation we will discuss what each of them are, how they look, and how they are created. As well as what it would be like to stand in one, like you are standing in a room full of mirrors. Then I will conclude with how we may be living in a universe that looks just like this, possibly, an infinitely large room of mirrors.

**Hugh McKenny, Hobart and William Smith Colleges***Optimizing Fairness in British Parliamentary Debates*

Abstract: It is commonly believed within the collegiate debate circuit that the current structure of debate tournaments is systematically flawed as some of the best teams frequently do not advance out of the preliminary rounds. In other words, debate tournaments, under the current structure, are bad at correctly ranking teams. We developed a computational discrete model to simulate British parliamentary debate tournaments. Through computationally intensive manipulation of various model parameters, we explored alternative tournament structures. To evaluate the correctness of various structures, we developed metrics for the fairness or accuracy of the resulting rankings. In this talk, we will outline what makes debate tournaments unlike other competitions, consider various ranking metrics to use with incomplete and nontransitive tournament outcomes, and highlight a couple of tournament structures that improve the fairness of debate tournaments.



**Molly Noel, Ithaca College***Online Change-Point Detection in the Mean of High-Dimensional Data*

Abstract: We develop a method of detecting change points in high-dimensional online data using means. A new stopping rule is proposed that relies on the spatial dependence of the data but does not assume the data follows a Gaussian distribution. We study the asymptotic properties of this new stopping rule. An explicit expression for the average run length (ARL) is derived when there is no change. When there is a change point, an upper bound is established for the expected detection delay (EDD) which demonstrates the impact of data dimensionality and dependence. Our method is applied to simulated data in order to verify its accuracy under a range of parameters. We apply our results to data collected in Beijing, measuring the level of pollutant PM2.5 in the atmosphere. This research was conducted under NSF grant DMS-1916239. *This project was undertaken as a collaboration between Olivia Beck (Colorado State University), Isabelle Hauge (University of Massachusetts Amherst), and Molly Noel (Ithaca College) with faculty advisor Jun Li (Kent State University).*

**Briana Palmer, The College at Brockport***A conjecture of George Miliakos*

Abstract: I will discuss a recent conjecture of George Miliakos concerning a relation between consecutive prime numbers. I will show counterexamples and will address some similar statements.

**Eric Piato, SUNY Geneseo***Critical Groups of Strongly Regular Graphs*

Abstract: Let  $G = (V, E)$  be a simple graph. The *critical group* (also called the *sandpile group*), denoted  $K(G)$ , is a finite abelian group associated with  $G$ . Concretely, viewing the Laplacian matrix  $L$  as a linear mapping  $\mathbb{Z}^{|V|} \rightarrow \mathbb{Z}^{|V|}$ , it turns out that  $\mathbb{Z}^V / \text{Im}(L) \cong \mathbb{Z} \oplus K(G)$ . In this talk, we discuss our results regarding the critical groups of strongly regular graphs  $\Gamma$ . In particular, we provide a complete characterization of  $K(\Gamma)$  under certain assumptions regarding the associated eigenvalues of  $\Gamma$ . In other cases, when the eigenvalues of  $\Gamma$  satisfy different (weaker) conditions, we are able to provide constraints on the form of the critical group. We conclude with a brief discussion regarding the question of existence of a strongly regular graph with given parameters, and explore how our work could be used to resolve this inquiry.

**Morgan Sherwood, The College at Brockport***Three squares in a circle*

Abstract: I will discuss a recent problem posted on the internet concerning 3 squares in a circle. I will show why the problem is wrong, how to fix it, and how to solve it.

**Nicolas van Kempen, SUNY Oswego**  
*Cocycle Invariant and Oriented Singular Knots*

Abstract: Finding an efficient way to compute whether or not two knot diagrams are representations of the same knot is one of the most researched problems in knot theory, with few efficient solutions. In this presentation, we will introduce a new way to compare knots diagrams, the cocycle invariant, which provides an enhancement of current methods to more easily differentiate topologically distinct knots. We will present algebraic structures such as quandles and singquandles, which will enable us to work with oriented singular knots. We will explain and provide examples of how these structures can be related to knot diagrams. We will then present the notion of a quandle cocycle invariant on oriented singular knots, defining precisely the invariant, giving a quick overview of the algorithm we had to develop for this project, and once more providing examples to confirm and illustrate the theory. While researching this invariant over the past summer, we have obtained many promising results, and are still working to further better the invariant.

**Nicole Zhe, The College at Brockport**  
*A Monotone Sequence Related to Prime Numbers*

Abstract: I will show that a certain sequence related to the sequence of prime numbers is increasing.

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