

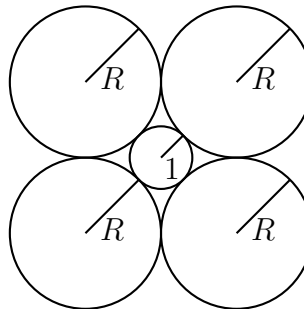
2019 Math Exploration Day

Team Competition

1. You use part of your summer earnings to buy several equally-priced gifts for your friends. If you use one fifth of your money to buy one third of the gifts, what fraction of your money remains after buying all of the gifts?

2. A student drove a compact car 240 miles from home to the beach, averaging 30 miles per gallon. On the trip back to home, the student drove his friend's SUV, which averaged 20 miles per gallon. What was the average gas mileage, in miles per gallon, for the student's round trip?

3. A circle of radius 1 is surrounded by 4 circles of radius R as below. What is the exact value of R ?



4. Define the operation \star by $x \star y = (x + y)y$. What is the value of $(6 \star 7) - (7 \star 6)$?

5. Express as a fraction in lowest terms: $\frac{1 + 2 + 3 + 4 + \cdots + 2019}{1 + 2 + 3 + 4 + \cdots + 2018}$

6. The point $(6, 2)$ is reflected across the line $x - y = 0$ and then rotated 90° clockwise about the origin. What are the coordinates of the transformed point?

7. For how many real numbers x is $\sqrt{169 - \sqrt{x}}$ a positive integer?

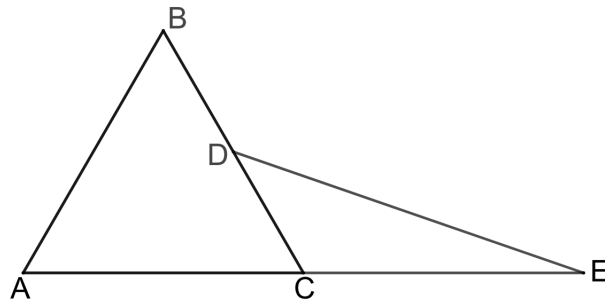
8. The area of a right triangle is 330 and its hypotenuse has length 61. What is the perimeter of the triangle?

9. What is the sum of every multiple of 3 from the list of whole numbers from 1 to 2019, inclusive?

10. The local animal shelter adopts out cats and dogs. Adoptions are 10% higher than last year. The number of dog adoptions increased by 5% and the number of cat adoptions increased by 20%. What fraction of this year's adoptions were cats?

11. Consider the line through the points $(1, 7)$ and $(6, 42)$. How many points on this line have the property that both coordinates are integers and the points lie inside the circle $x^2 + y^2 = 2500$?

12. Equilateral triangle $\triangle ABC$ has side length 4, D is the midpoint of \overline{BC} , and C is the midpoint of \overline{AE} . What is the area of the triangle $\triangle CED$.



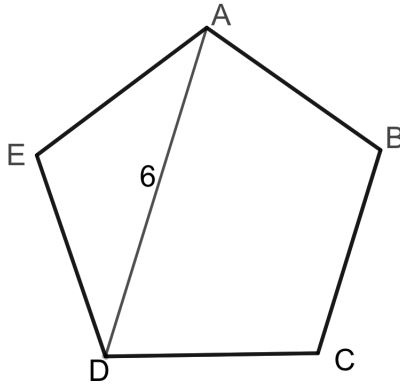
13. How many palindromes are there between 10,000 and 100,000? A palindrome is a number that reads the same forward and backward, such as 121.

14. Two different numbers are chosen at random from the set $\{3, 8, 9, 14, 17, 22, 31, 34, 40, 51\}$. What is the probability that the sum of the two numbers is odd?

15. You drop a rubber ball from a height of 10 meters. Each time the ball strikes the ground it bounces straight back up exactly half the distance of the height it just fell. If your friend catches the ball at the peak after the fifth bounce, what is the total distance that the ball travelled?

16. Solve for x : $\log(\log 5) + \log x = \log(\log 625)$

17. Given regular pentagon $ABCDE$, connect the midpoints of consecutive sides with line segments to create another pentagon inside the original. If the diagonal AD has length 6, what is the perimeter of the interior pentagon?



18. The first term in my sequence is 16. The next term in the sequence is the sum of the square of each digit in the previous terms; that is, the second term is $1^2 + 6^2$. Each of the remaining terms in my sequence is the sum of the squares of the digits of the previous term. What is the 2019th number in my sequence?

19. What digit (1-9) must go in the highlighted cell of the following Sudoku puzzle?
 (Reminder: In a Sudoku puzzle, the digits 1-9 must appear exactly once in each row, in each column, and in each 3×3 block indicated by thick lines.)

		4			6		2	
		7	8			9	1	
						3		8
	1	8	3			2		
3		?	7	8	9			1
					1		6	
8		3				5		
	4	5			3	6		
	2	6	5			1		

20. A point P is chosen at random from the rectangular region with vertices $(0, 0)$, $(2, 0)$, $(2, 1)$, and $(0, 1)$. What is the probability that P is closer to the origin than it is to the point $(2, 2)$?