

2022 Math Exploration Day

Team Competition

1. Megan worked out on a treadmill for 15 minutes, walking at 4.2 miles per hour for part of the time and running at 5.4 miles per hour for the remainder of the time. If she traveled a total of 1.3 miles, how long did she spend walking?

2. You roll two standard, six-sided dice and you subtract the smaller number from the larger number. If you roll the same number on each die, it doesn't matter the order in which you subtract. What is the most likely difference to occur?

3. Olivia raises chickens and cows. She has 54 animals with a total of 122 legs. How many chickens does she have?

4. Triangle ABC has coordinates $A = (4, 0)$, $B = (0, 4)$, and C is a point on the line $x + y = 10$. What is the area of triangle ABC ?

5. Define the operation \odot by $x \odot y = y^2 - x$. What is the value of $(3 \odot 5) \odot (2 \odot 4)$?

6. How many integers from 1 to 100 contain 5 as one of its digits but not a 3?

7. A train leaves Ithaca at 6:00am heading north at 40 miles per hour. At 7:00am, another train leaves Ithaca heading north on a parallel track at 50 miles per hour. At what time are both trains the same distance away from Ithaca?

8. Suppose that $19m$ is a perfect square. What is the smallest value of m such that $19+m$ is a perfect square.

9. What is the sum of all the roots of $(3x - 2)(x + 5) + (2x + 3)(3x - 2) = 0$?

10. If $12 \cos(x) = 5 \sin(x)$, compute $\sin(2x)$. Express your answer as a quotient of two integers.

11. A box contains either a white ball or a black ball. A black ball is then added to the box. You draw a ball from the box and it is black. What is the probability that the remaining ball is also black?

12. A motorcycle dealer is trying to sell a used bike and offers it on sale for 10% off the original price. When no one buys it, he offers 25% off the sales price. Again, no one buys it, and so he offers another 15% off of the second sales price. The bike sells for \$1836. What was the original price of the bike?

13. What is the smallest number that is divisible by 1, 2, 3, 4, 5, 6, 7, 8, and 9?

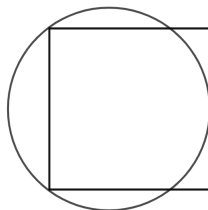
14. A line with slope 5 intersects a line with slope -2 at the point $(10, 20)$. How far apart are the x -intercepts of these lines?

15. I have \$4.65 in quarters and dimes. If the dimes were quarters and the quarters were dimes, then I would have \$1.20 more. How many quarters do I have?

16. For real numbers a and b , we have $\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}} = 2020$. What is the value of $\frac{a+b}{a-b}$?

17. Consider the triangle $\triangle ABC$ in the first quadrant formed by $A = (0, 0)$ and the points B, C which are the intersections of the coordinate axes with the line $x + 2y = 9$. How many points (x, y) are on or inside $\triangle ABC$ for which both coordinates are integers?

18. In the figure below, the square is tangent to the circle and intersects the circle in four other points. If the perimeter of the square is 32, what is the area enclosed by the circle?



19. What is the sum of the first 2022 odd numbers?

20. The first term in a sequence is 31. The next term in the sequence is the sum of the cube of each digit in the previous term; that is, the second term is $3^3 + 1^3 = 28$. Each of the remaining terms in my sequence is the sum of the cubes of the digits of the previous term. What is the 2022nd number in this sequence?