Sample Math Placement Exam Questions

This review is not intended to cover all of the material on the Math Placement Exam. Material on the Math Placement Exam that is not covered in this review includes:

- 1. Quadratic Functions
- 2. Exponential and Logarithmic Functions
- 3. Trigonometric Functions
- 4. Analytical trigonometry

This review is designed to be a refresher of elementary mathematics. Some students will find this material very easy and therefore may not need to study this material. Others may find material that they have never learned. This review is not intended to teach students material that they have never been taught.

The Math Placement Exam is intended to find out what the student knows and what they don't know so that they can be placed in the proper mathematics class. Therefore, if there is material that you are not familiar with, don't worry. You will be placed into a math class which will teach you that.

1 Order of Operations

The order in which we perform mathematical operations is:

- 1. Parenthesis: working from inside out
- 2. Exponents
- 3. Multiplication/Division: working from left to right
- 4. Subtraction/Addition: working from left to right

Example 1: $[2 \times (1-5) - 3] + 4$ First we subtract 5 from 1 and get -4. Next we multiply 2 by -4 and get -8. Then we subtract 3 from -8 and get -11. Finally we add 4 to -11 and get -7.

Example 2: $15 \div 5x3 - (3 + 2 \times 4)$ First we multiply the 2 by 4 and get 8. Next, add 3 to 8 and get 11. Then, divide 15 by 5 and get 3. After that, multiply 3 by 3 and get 9. Finally, subtract 11 from 9 and get -2.

Example 3: $12 + 4 \div 2 - 3$ First, divide 4 by 2 and get 2. Next, add 12 to 2 and get 14. Finally, subtract 3 from 14 and get 11.

Problem Set 1

1. Evaluate each of the following:

(a) $3 + 5 \times 7$	(i) $-7-4$
(b) $(3+5) \times 7$	(j) $-3+5$
(c) $3 \times (5 \times 7 + 8)$	(k) $4 - 11$
(d) $2 + ((5+7) \times 8) - 1$	(l) $-2 + (-6)$
(e) $16 \times 2 \div 8 - 1$	(m) $3 - (-7)$
(f) $16 - 3 + 4 \times 2$	(n) $-3 - (-4)$
(g) $16 - (3 + (4 \times 2))$	(o) $-2 - (+3)$
(h) $16 - (3+4) \times 2$	

2. Place parentheses in $25 - 5 \times 4 - 2$ to make it equal:

- (a) 40
- (b) 78
- (c) 3

3. Place parentheses in $36 \div 6 \times 3 - 1 + 2$ to make it equal:

(a) 3	(d) 19
(b) 5	(e) 0
(c) 14	(f) undefined

2 Fractions

Addition/Subtraction: In order to add or subtract fractions they must have a common denominator. A common denominator can always be obtained by multiplying all of the denominators together. For example, if we are adding 1/4 to 1/9 a common denominator would be 4×9 or 36. After a common denominator is created we then add/subtract the numerators.

Example 1: 1/4 + 1/9 becomes 9/36 + 4/36 which equals 13/36.

Example 2: 2/3 - 3/7 becomes 14/21 - 9/21 which equals 5/21.

Note: We do not have to have the lowest common denominator but you should express your answers in reduced form.

Example 3: 3/4 + 5/6 can be done as 18/24 + 20/24 which equals 38/24. This can be reduced to 19/12.

This example also can be done as 9/12 + 10/12 which also equals 19/12.

Multiplication: You don't need common denominators to multiply two fractions. When you multiply two fractions you multiply the numerators together and the denominators together.

Example 4: $4/5 \times 2/3$ equals 8/15.

Division: When you divide two fractions you take the reciprocal of the fraction you are dividing by and multiply.

Example 5: $3/5 \div 2/3$ is $3/5 \times 3/2$ which equals 9/10.

Example 6: $\frac{2/3}{3/4}$ This is actually a division problem. It is the same as $2/3 \div 3/4$ which is $2/3 \times 4/3$ which equals 8/9.

Example 7: $2\frac{3}{4} + 1\frac{7}{8}$ This is easier if we change the mixed numbers to fractions. So we have 11/4 + 15/8. A common denominator is 8 so the problem becomes 22/8 + 15/8 which equals 37/8.

Problem Set 2

Perform indicated operations. Reduce all answers to lowest terms.

1. $3/4 - 5/11$	9. $7/40 + 6/60 + 1/20$
2. $5/9 + 1/12$	10. $2/3 \times 3/4$
3. $7/10 - 1/5$	11. $3/4 \div 5/7$
4. $13/19 + 1/38$	12. $12/3 \times 23/4$
5. $5/16 + 3/8$	13. $3/4 \div 4/5 \times 1/7$
6. $8/9 - 2/27$	14. $23/4 + 17/8 \times 3/7$
7. $9/12 + 3/4$	15. $1/2 + 4/5 \times 5/8 - 2/3$
8. $11/24 - 5/72$	

Problem Set 3

- 1. Mike jogged around the park 3 and 1/2 times. Once around the park is 3/4 miles. How far did he jog?
- 2. The contents of a container that holds 7/8 of a liter is separated into six equal parts. How much is there in each part?
- 3. The contents of a bottle that is two-thirds full weigh 3 1/2 grams. What do the contents of the bottle weigh when the bottle is full?
- 4. One container contains 16 ounces of coffee. Another container contains 16 ounces of milk. One ounce of milk is poured into the coffee and then the mixture is stirred. After that one ounce of the stirred mixture is then poured back into the milk container. Now, is there more milk in the coffee container or more coffee in the milk container? Justify your answer.
- 5. One day Jill spends 1/3 of the day sleeping, 1/6 of the day in school, 1/4 of the day studying, 1/12 of the day eating and the rest goofing-off. How much of the day did Jill goof-off?
- 6. Find the perimeter of a triangle, whose sides are 3/8 m, 7/16 m and 5/8 m.

3 Percentages

Changing percentages to decimals: To change a percentage to a decimal you move the decimal two places to the left.

For example, 12% = 0.12; 21.7% = 0.217; 2.3% = 0.023; 124% = 1.24.

Changing decimals to percentages: To change a decimal to a percentage you move the decimal two places to the right.

For example, 0.14 = 14%; 2.17 = 217%; 0.015 = 1.5%; 0.1234 = 12.34%.

Finding a percentage: To what percentage one number, x, is of another, y, divide x by y and multiply by 100. For example, to find what percent 30 is of 40 divide 30 by 40 obtaining 0.75 and multiply the result by 100 resulting in 75%.

Other problems involving percentages: There are two other types of problems involving percentages. The first is: "What is p% of b?" To solve this problem you would set up a ratio x/b = p/100, where x is what we are solving for. Then we would solve for x.

Example 1: What is 40% of 30? First we set up the ratio x/30 = 40/100. Next we multiply both sides by 30 resulting in x = 1200/100 or 12.

There is another way to solve this problem. Just multiply 30 by 40% (or 0.4) to yield the answer. The second type of problem is "p% of what is a?" To solve this problem we use the ratio a/x = p/100, again where we are solving for x. Then we cross multiply and solve for x.

Example 2: 20% of what is 45? First we set up the ratio

$$45/x = 20/100.$$

Then we cross multiply resulting in

4500 = 20x.

Next we divide by 20 giving us 225. This is the answer.

Notice the difference between the two types of problems.

Percent increase/decrease: To find a percent increase/decrease first you subtract the two numbers to find the amount of increase/decrease. Then you put this result over the *starting amount* and multiply by 100.

Example 3: 20 is increased to 36. The amount of increase is 36 - 20 = 16. The percent increase is 16/20 = 0.80 = 80%.

Example 4: 36 is decreased to 20. The amount of decrease is 36 - 20 = 16. The percent decrease is 16/36 = 0.444 = 44.4%.

Problem Set 4

- 1. What is 48% of 350?
- 2. 16 is what percent of 60?
- 3. Find 1.3% of 420.
- 4. 12 is 40% of what number?
- 5. Bill got 22 out of 35 problems correct on a test. What percent right did he get?
- 6. Trisha works on a 15 percent commission on all the items she sells. Last week she made \$365. What was the total amount of her sales?
- 7. The population of a city increased from 140,000 to 148,000. What is the percent increase?
- 8. A table that originally sold for \$92 is on sale at 35% off. What is the sale price?
- 9. I work at a store where I get 15 percent off everything. The store is having a sale where everything is 20 percent off. How much will I pay for a \$200 coat? What percent off did I get?

4 Solving Single Variable Equations

In order to solve a single variable equation:

- 1. Put terms with variable on one side of the equation and terms without the variable on the other side of the equation.
- 2. Divide both sides by the coefficient of the variable.

Example 1: Solve 8x + 7 = 2x - 9 for x. First subtract 2x and 7 from each side resulting in 6x = -16. Next divide both sides by 6 giving us the answer x = -16/6 which reduces to x = -8/3.

Example 2: Solve 2xy - 3ay = 6xa - 4by for x. First add 3ay and subtract 6xa to both sides yielding 2xy - 6xa = 3ay - 4by. Next factor x from the left side of the equation resulting in

$$x(2y - 6a) = 3ay - 4by.$$

Finally divide both sides by 2y - 6a giving us the answer x = (3ay - 4by)/(2y - 6a).

Problem Set 5

Solve each of the following for x.

	2x + 4 = 3(x - 1)	10. $\frac{1}{1+1/x} = 2$
2.	$3-x = \frac{2x+1}{2}$	11. $\frac{x-1}{x+1} = 1$
•	ax + b = 0 6(x + c) = 5	12. $\frac{x+1}{3} + 1 = 11 - x$
	ax + b = c	$\frac{3}{13. \frac{12}{3} - 1 = 5}$
	$V = 1/2pa^2x$	$\begin{array}{c} x \\ 14. \frac{3+x}{3+x} = 1 \end{array}$
•••	$\frac{3+x}{x} = 2$	14. $\frac{x}{x} = 1$ 15. $4/3(x-1) = 3$
8.	$\frac{2x-1}{x-1} = 3$	16. $4/3(x-1) = x$
9.	$\frac{2x+1}{x-1} = 2$	

5 Solving word problems in one variable

- 1. Determine which quantity you are being asked to find. Choose a letter to represent this unknown quantity. Write down exactly what this letter represents.
- 2. Write the word problem as an equation.
- 3. Solve the equation for the unknown quantity.
- 4. Answer the question asked.

Example 1: A car rental agency charges \$240 per week plus \$0.19 per mile. How far can you travel on a budget of \$500.

Let m = number of miles you travel

240 + 0.19m = 500m = 260/0.19 = 1368.4 miles

Problem Set 6

- 1. If 6 times a number is increased by 10 the result is 8 times the number, less 12.
 - (a) Let the number be x. What expression represents 6 times the number, increased by 10? Eight times the number, less 12?
 - (b) Translate the problem into a mathematical equation.
 - (c) Find the number.
- 2. One number is 5 more than twice another number. Their sum is 80. Find the numbers.
- 3. A student has scores of 61 and 77 on the first two exams. What must her score on the third exam be if the three-exam mean is to be 75? Hint: The mean is their sum divided by 3.

- 4. A father is three times as old as his son, but 15 years from now he will be only twice as old as his son.
 - (a) Let x designate the son's age now. What is the father's age now?
 - (b) What will be the age of the son in 15 years? The father? Express the fact that the father is (in 15 years) twice as the son.
 - (c) How old is the son now?
- 5. Rachel is 3 years younger than Joel. Forty-one years ago, she was two-thirds his age. How old is Rachel?
- 6. While filing out your income tax, you suddenly realize that you can deduct the sales tax on a boat you bought last summer. You find the canceled check, which is made out for \$2588.60. You remember that this includes \$7.50 for a license. If the sales tax in your state is 6%, how much tax did you pay?
- 7. Going at a certain speed it took Mary 8 hours to get to Center City. If she could have driven 10 miles an hour faster, it would only have taken 6 2/3 hours. How far did she drive?
- 8. John can mow a yard in 6 hours and Amy can mow the yard in 4 hours. How long will it take if they work together?
- 9. Susan's grandmother is 11 times as old as Susan now, but in 45 years she will be only twice as old. What are their ages?
- 10. John drives 20,000 miles each year and gas costs \$1.80 per gallon. His present car gets 20 miles per gallon. If he bought a new car, he figures he would save \$360 per year on gas. How many miles per gallon does the new car get?
- 11. How pure acid must be added to a 10 gallon solution which is 40 percent acid to obtain a solution which is 60 percent acid?
- 12. How much solution which is 80 percent acid would have to be added to a 10 gallon solution which is 40 percent acid to obtain a solution which is 60 percent acid?

6 Inequalities

Solving inequalities is similar to solving equalities except . . . when you multiply/divide by a negative number, you must reverse the sign of the inequality.

Example 1: Solve $8x + 7 \le 2x - 9$ for x. First subtract 2x and 7 from each side resulting in $6x \le -16$. Next divide both sides by 6 giving us $x \le -16/6$ which reduces to $x \le -8/3$.

Example 2: Solve $7 - 2x \ge 10$ for x. First subtract 7 from each side resulting in $-2x \ge 3$. Next divide each side by -2. Since we are dividing by a negative number we must change the sign of the inequality. Thus we obtain $x \le -3/2$.

Example 3: Solve $7 \le 2x - 4 \le 12$ for x. First break the problem up into two parts, $7 \le 2x - 4$ and $2x - 4 \le 12$. Now work on each inequality separately. Taking $7 \le 2x - 4$, add 4 to each side giving us $11 \le 2x$. Now divide by 2 resulting in $11/2 \le x$. Do the same operations to the other inequality; i.e., add 4 to each side then divide by 2 giving us $x \le 8$. The answer to the problem is $11/2 \le x \le 8$.

Problem Set 7

Solve for x and graph the solution:

1.
$$\frac{x}{3} - 1 < -2$$
10. $x - 5 < 6$ and $2x - 1 > 9$

2. $-5x + 12 \le 17$
11. $5x + 6 > -4$ and $4 - x < 8$

3. $\frac{2x - 9}{3} > 5$
12. $4 - 3x > 13$ and $2x - 3 > -11$

4. $3x - (x - 5) \le x + 7$
13. $2x - 4 < 0$ and $2x - 3 > 3$

5. $\frac{x}{4} - \frac{2x}{3} \ge \frac{1}{12}$
14. $3 \le x - 5 \le 5$

6. $\frac{3x - 1}{5} > \frac{4x - 3}{2}$
15. $-6 < 3x < 9$

7. $5x - 8 > 7x - 12$
16. $1 \le 2x + 1 \le 7$

8. $\frac{x}{5} - \frac{5x}{3} \ge \frac{4}{15}$
18. $1 \le 2 - x < 4$

9. $2x - 8 < 0$ and $2x - 1 \ge 1$
12

7 Solving Systems of Equations

The most used method to solve systems of equations is to get the equations in a format such that when you add the two equations that one of the variables is eliminated.

Example 1:

$$\begin{array}{rcl} 2x + 3y &=& 4\\ x - y &=& 7 \end{array}$$

If we multiply the second equation by 3 we would have:

$$\begin{array}{rcl} 2x+3y &=& 4\\ 3x-3y &=& 21 \end{array}$$

Adding the two equations results in 5x = 25; so x = 5. We now substitute 5 for x into either of the two equations and we get 5 - y = 7; so y = -2. Thus the solution is x = 5 and y = -2.

Example 2:

$$2x + 3y = 9$$
$$3x - 2y = 7$$

This time we need to multiply the top equation by 2 and the bottom one by 3 resulting in:

$$4x + 6y = 18$$
$$9x - 6y = 21$$

Adding the two equations together gives us 13x = 39, so x = 3. Substituting 3 for x in the top equation gives us 6 + 3y = 9 or y = 1.

Example 3:

$$\begin{array}{rcl} x+y &=& 4\\ 3x+3y &=& 2 \end{array}$$

If we multiply the top equation by -3 we get:

$$\begin{array}{rcl} -3x - 3y &=& -12\\ 3x + 3y &=& 2 \end{array}$$

Adding the two equations gives us 0 = 10. Since this is never true we see that there is no solution.

Problem Set 8

Solve the following systems of equations. Be sure to check your answer in both equations.

7. $x - 2y = 8$
3x + y = 3
8. $5x + 2y = 10$
3x - 4y = 6
9. $5x + 2y = 1$
3x + y = 2
10. $3x + 4y = 3$
2x - 3y = 2
11. $4x + 3y = 0$
6x + 6y = 1
12. $2x + 3y = -9$
3x + 2y = -6

Problem Set 9

- 1. The price of admission for a high school play is \$1.50 for students and \$2.25 for non-students. If 450 tickets were sold for a total of \$777.75, how many tickets of each kind were sold?
- 2. New West Airlines flies from LA to Dallas with a stopover in Phoenix. The airfare to Phoenix is \$45 while the fare to Dallas is \$60. A total of 185 passengers boarded the plane in LA and fares totaled \$10,500. How many passengers got off in Phoenix?
- 3. A merchant wishes to mix peanuts costing \$2.00 per pound with cashews costing \$3.50 per pound to obtain 60 pounds of a mixture costing \$2.65 per pound. How many pounds of each variety should be mixed?
- 4. A stationary company sells two types of notebooks to college bookstores, the first wholesaling for 50 cents and the second for 70 cents. The company receives an order for 500 notebooks, together with a check for \$286. If the order fails to specify the number of each type, how should the company fill the order?
- 5. In the barnyard is an assortment of chickens and pigs. Counting heads I get 13; counting legs I get 46. How many pigs and how many chickens are there?
- 6. On a 24-item true-false test, Professor Witquick gave 5 points for each correct response but took off 7 points for each wrong one. Homer answered all the questions and came up with a big fat zero for his score. How many did he get right?
- 7. We have two solutions of acid. One is pure acid and the other is 30% acid. How much of each is needed to obtain 12 gallons of a solution that is 50% acid?
- 8. I have two containers of antifreeze. One is 40% antifreeze and the other is 70% antifreeze. How much of each is needed if I want 14 gallons of 60% antifreeze?

8 Graphing

Finding the equation of a line: One form of equation of a line is y = mx + b, where m is the slope of the line and b is the y-intercept. The slope of the line is the angle and direction of the line:

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

The y-intercept is the point at which x = 0.

Example 1: Find the equation of the line passing through the points (4, 2) and (3, 6). First we find m: m = (6-2)/(3-4) = -4. Then we plug either point into the equation y = -4x+b and solve for b. If we plug in the point (4, 2) we would get

$$2 = -4(4) + b$$

so b = 18. So the equation is

$$y = -4x + 18.$$

Problem Set 10

Find the equation of the straight line passing through each of the following pairs of points:

1. $(1,0)$ and $(2,3)$	5. $(1/2, 2)$ and $(2, -1)$
2. $(1,3)$ and $(2,1)$	6. $(-0.5, 10)$ and $(0, 0)$
3. $(1, -1)$ and $(1/2, 2)$	7. $(0,0)$ and $(1,2)$
4. $(0,5)$ and $(8,10)$	8. (a, b) and $(1, 3)$

Problem Set 11

Find the equation of the line with the following properties and graph the line.

- 1. Slope = 4, passing through (2,3)
- 2. Slope = -2, passing through (-4, 5)
- 3. Slope = -1, *x*-intercept is 6
- 4. x-intercept is 3 and y-intercept is 2
- 5. no slope, passing through (2,3)
- 6. Slope is 0, passing through (5,3)
- 7. Slope is 1/2, passing through (1, -5)

9 Exponents

Rules for exponents:

1. $x^m x^n = x^{m+n}$ 2. $(x^m)^n = x^{mn}$ 3. $x^m/x^n = x^{m-n} = 1/x^{n-m}$ 4. $(xy)^m = x^m y^m$ 5. $x^{-n} = 1/x^n$

6.
$$x^0 = 1$$

Example 1: $a^2a^4 = a^6$ Example 2: $(x^3)^4 = x^{12}$ Example 3: $x^3/x^5 = x^{-2} = 1/x^2$ Example 4: $(xy)^3 = x^3y^3$ Example 5: $x^{-2} = 1/x^2$ Example 6: $(x^2y^4)^2 = x^4y^8$ Example 7: $(x^{-2}y^4)^3 = y^{12}/x^6$ Example 8: $(x^3y^4)^0 = 1$

Problem Set 12

Simplify each of the following. Write the answer without negative exponents.

1. x^4x^3	9. $(6/x^2)^3$
2. y^3y	10. $(5x^2/y)^2$
3. x^4/x^2	11. $(x^{-4})^2$
4. $(3x)^0$	12. x^4/x^{-3}
5. $(x^5)^2$	13. $(y^{-1})^{-1}$
6. $(-x^2)^4$	14. $\frac{1}{x^{-4}}$
7. $(3x^2)^3$	2^{-2}
8. $(xy)^3$	15. $\frac{2}{2^3}$

Problem Set 13

Simplify each of the following. Write the answer without negative exponents.

1.	$rac{x^2y^6}{x^4y}$	7.	$\frac{12x^{-4}y}{24x^{-3}y}$
2.	$\frac{5x^{12}y^2}{10xy^8}$		$(-4x^{-3}y^2)(-2x^4y^5)$ $(-3x^{-2}y^{-2})(2x^4y^6)$
3.	$\frac{5xy^5}{2x^{-3}y^8}$	10.	$(4x^3y^{-2})^{-3}$
4.	$(3x^2y)(-2x^4y^6)$		$(-2xy^{-5})^{-3}$
5.	$(2x^{-4})(3x^6)$	12.	$\frac{x^{-4}y^{-2}}{z^2}$
6.	$(2x^{-4}y^{-5})(x^4y^5)$		

10 Answer Key

Problem Set 1

(f) 21 (g) 5 (h) 21. (a) 38 (b) 56 (c) 129 (d) 97 (e) 3 (i) -11(j) 2 (l) - 8(n) 1 (o) -5 2. (a) (25-5)(4-2) (b) $(25-5) \times 4-2$ (k) - 7(m) 10 (c) $25 - 5 \times 4 - 2$ 3. (a) $36 \div (6 \times 3) - 1 + 2$ (b) $36 \div (6 \times (3 - 1)) + 2$ (c) $36 \div 6 \times (3 - 1) + 2$ (d) $36 \div 6 \times 3 - 1 + 2$ (e) $36 \div 6 \times (3 - (1 + 2))$ (f) $36 \div (6 \times (3 - (1 + 2)))$

Problem Set 2

Problem Set 3

1. 21/8 miles 2. 7/48 liter 3. 21/4 grams 4. the same 5. 1/6 6. 23/16 m

Problem Set 4

1. 168 2. 26.7% 3. 5.46 4. 30 5. 62.9% 6. \$2433.33 7. 5.7% 8. \$59.80 9. (a) \$136 (b) 32%

Problem Set 5

1. 7 2. 5/4 3. -b/a 4. (5-6c)/6 5. (c-b)/a 6. $2V/(pa^2)$ 7. 3 8. 2 9. no solution 10. -2 11. no solution 12. 7 13. 2 14. no solution 15. 13/4 16. 4

Problem Set 6

Problem Set 7

Problem Set 8

Problem Set 9

Problem Set 10

1. y = 3x - 3 2. y = -2x + 5 3. y = -6x + 5 4. y = (5/8)x + 5 5. y = -2x + 36. y = -20x 7. y = 2x 8. y = (-b/a)x

Problem Set 11

1. y = 4x - 5 2. y = -2x - 3 3. y = -x + 6 4. y = (-2/3)x + 2 5. x = 2 6. y = 37. y = (1/2)x - 11/2

Problem Set 12

Problem Set 13