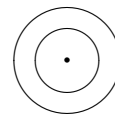


Sample Mathematics Placement Exam

1. What is the area in square inches of the region between two concentric circles of radii 5 inches and 3 inches? See figure.

- (a) 2π (b) 4π (c) 9π (d) 16π (e) 25π



Answer:

In order to answer this question you need to know that an area of a circle is πr^2 where r is the radius of the circle.

The second thing you need to understand is how to find the area between two regions where one includes the other. The answer is simply the difference between the areas of the regions.

Thus the area of the regions between the two circles is $25\pi - 9\pi = 16\pi$ inches.

2. An article usually sells for \$8.00 but is on sale at 20% off. If there is a sales tax of 5%, the total cost to the buyer of the article is

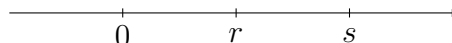
- (a) \$6.00 (b) \$6.08 (c) \$6.40 (d) \$6.72 (e) \$6.80

Answer:

20% of \$8 is $0.2 \cdot \$8 = \1.60 so the sale price before tax was \$6.40. The sales tax was 5% of \$6.40 = \$0.32 so the total was $\$6.40 + \$0.32 = \$6.72$

3. Let two numbers r and s be located on a number line as shown below. Which one of the following is true?

- (a) $r > s > 0$ (b) $s < 0 < r$ (c) $0 > s > r$
(d) $0 > r > s$ (e) $0 < r < s$



Answer:

You just need to know what $<$ and $>$ mean. “ $5 < 7$ ” means “5 is less than 7” and “ $7 > 3$ ” means “7 is greater than 3.” The line segment above is a number line and the numbers increase left to right so the options mean: (a) r is bigger than s which is bigger than 0 (false). (b) s is less than 0 which is less than r (false) (c) 0 is bigger than s which is bigger than r (false) (d) 0 is bigger than r which is bigger than s (false) (e) 0 is less than r which is less than s (true).

4. If $\frac{4}{3}x - \frac{1}{2} = 0$, then $x =$

- (a) $\frac{3}{8}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{11}{6}$ (e) $\frac{8}{3}$

Answer:

Since we know that $\frac{4}{3}x - \frac{1}{2} = 0$ we can add $\frac{1}{2}$ to both sides of the equation and get that $\frac{4}{3}x = \frac{1}{2}$. We next multiply both sides by 3 and get $4x = \frac{3}{2}$. Finally we divide by 4 and get the answer: $x = \frac{3}{8}$.

5. $-5[4a - (-3)(2a)] =$
(a) $-70a$ (b) $-50a$ (c) $-10a$ (d) $5a$ (e) $10a$

Answer:

To answer this question you need to know that $-(-x) = +x$ for any number x . It is like mirroring a picture twice and getting back to the original. For example $-(-5) = +5$. Thus $4a - (-3)(2a) = 4a + 3(2a)$. Next you need to know that $3(2a)$ means 3 times $2a$ which is $6a$. So this expression becomes $4a + 6a = 10a$. Finally we multiply by -5 and get the answer $-50a$.

6. $(xy^3)^2 =$
(a) x^2y^3 (b) x^2y^6 (c) $(xy)^6$ (d) x^2y^5 (e) $(xy)^5$

Answer:

You need to know that x^2 means xx (x times x or $x \cdot x$) no matter what x is. So $(xy^3)^2 = (xyyy)(xyyy) = xyyyxyyy = xxyyyyyy = x^2y^6$.

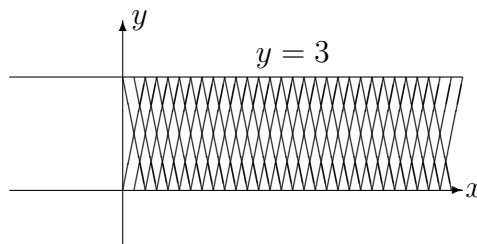
7. $\sqrt{3}(\sqrt{3} + 2) =$
(a) $9 + 2\sqrt{3}$ (b) 5 (c) $3 + 2\sqrt{3}$ (d) $\sqrt{3} + 2\sqrt{3}$ (e) 11

Answer:

Since $\sqrt{3}$ is a number such that $\sqrt{3}\sqrt{3} = 3$, the answer is $3 + \sqrt{3}2 = 3 + 2\sqrt{3}$.

8. The shaded area (including the boundary) represents the graph of which of the following sets of inequalities?

- (a) $x \leq 3$ and $y \leq 3$
(b) $x \leq 3$ and $y \geq 0$
(c) $0 \leq x \leq 3$ and $y \geq 0$
(d) $y \leq 3$ and $x \geq 0$
(e) $0 \leq y \leq 3$ and $x \geq 0$



Answer:

The answer is $0 \leq y \leq 3$ and $x \geq 0$ (x is not “stopping” at any point).

9. $\frac{x-3}{8} - \frac{7}{4} = \frac{5}{8}$ has a solution of
(a) -12 (b) -6 (c) 15 (d) 16 (e) 22

Answer:

We can start by adding $7/4$ to both sides of the equation. We get the equation

$$\frac{x-3}{8} = \frac{19}{8}.$$

Then multiply both sides by 8 and get

$$x - 3 = 19.$$

Finally, add 3 to both sides to get $x = 22$.

10. $\frac{x}{5y} \div \frac{2x}{3y} =$

- (a) $\frac{3}{10}$ (b) $\frac{2x^2}{15y^2}$ (c) $\frac{x}{15y}$ (d) $\frac{10}{3}$ (e) $\frac{x+3y}{2x+5y}$

Answer:

For this question you need to know that $(x/5y)/(2x/3y)$ is the same as $(x/5y) \times (3y/2x)$ (by the rule of reciprocity, or “invert and multiply”). The rest is easy. Cancel out the x 's and y 's to get $3/10$.

11. If $f(x) = \frac{x+3}{5-x}$, then $f(a+4) =$

- (a) $\frac{a+7}{1-a}$ (b) $\frac{a+7}{9-a}$ (c) $\frac{38-6a}{5-a}$ (d) $\frac{23-3a}{5-a}$ (e) $\frac{a+7}{5-a}$

Answer:

You need to substitute $a+4$ for x getting $f(a+4) = (a+4+3)/(5-(a+4))$. Therefore, $f(a+4) = (a+7)/(1-a)$.

12. If $x^2 + 2x = 3$, then x could equal

- (a) -3 (b) -2 (c) -1 (d) 0 (e) 3

Answer:

Subtract 3 from both sides:

$$x^2 + 2x - 3 = 0.$$

Factor the left-hand side:

$$(x+3)(x-1) = 0.$$

The two solutions are $x = -3$ and $x = 1$.

13. $\sqrt[6]{a^2b^3} =$

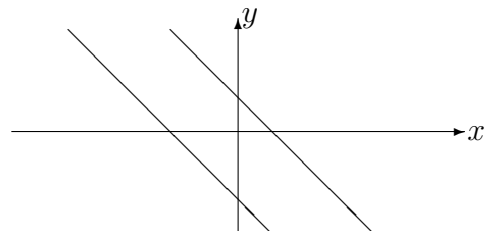
- (a) $a^{12}b^{18}$ (b) $a^{-4}b^{-3}$ (c) $a^{1/3}b^{1/2}$ (d) a^4b^3 (e) None of above

Answer:

Here you need to know that, $\sqrt[6]{x} = x^{1/6}$. Thus $\sqrt[6]{a^2b^3} = a^{2/6}b^{3/6}$. This reduces to $a^{1/3}b^{1/2}$.

14. The two parallel lines represent the graphs of which of the following pair of equations?

- (a) $x - 2y = 3$ and $x - 2y = 7$
(b) $x + y = 1$ and $x + y = -2$
(c) $x + y = 3$ and $2x + 2y = 6$



- (d) $x + y = 3$ and $x - y = 5$
(e) $x - y = 7$ and $x - y = 14$

Answer:

Here you need to know the basic equation of a line, $y = mx + b$. The value of b is the y -intercept and m is the slope. The two lines in question both have (the same) negative slope; one has a positive y -intercept, and the other has a negative y -intercept. Choice (b) has a pair of lines with positive and negative values for b and the same negative slope.

15. For any x , $|x - 7| =$
(a) $x - 7$ (b) $|7 - x|$ (c) $|x + 7|$ (d) $-x - 7$ (e) $|-(x + 7)|$

Answer:

Recall that $|x - a|$ is the distance between x and a . Clearly the distance between x and 7 is the same as the distance between 7 and x , so (b) is the correct answer.

16. $\log_6 4 + \log_6 9 =$
(a) 2 (b) $\log_6 13$ (c) $\frac{13}{6}$ (d) 78 (e) $\log_6(\frac{4}{9})$

Answer:

You need to know the properties of logarithms for this question. One property is that the log of a product is the sum of the logs, so $\log_6 4 + \log_6 9 = \log_6(36)$. Then notice that $36 = 6^2$. Therefore, $\log_6 36 = \log_6(6^2)$. The fundamental property of logarithms is that a logarithm is an exponent. (That is, $\log_b a$ is the number you raise b to in order to obtain a .) So $\log_6(6^2) = 2$.

17. Given a rectangle with sides of length x and width y . Suppose the length x is doubled and the width y is halved. The new perimeter is
(a) $4x + y$ (b) $(2x)(\frac{y}{2})$ (c) $2x + \frac{y}{2}$ (d) $x^2 + y$ (e) $x^2 + \frac{y}{2}$

Answer:

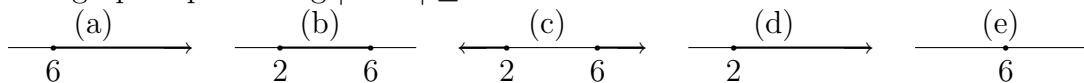
Here you need to know that the perimeter of a rectangle is equal to $2(x + y)$. If x is doubled and y is halved then perimeter of the new rectangle is equal to $2(2x + \frac{1}{2}y)$ which simplifies to $4x + y$.

18. If $\log_2 16 = 8 \cdot 2^{-x}$, then $x =$
(a) -7 (b) $-\frac{4}{3}$ (c) -1 (d) $-\frac{2}{3}$ (e) 1

Answer:

Recall again the properties of logarithm functions. Notice also that $16 = 2^4$ and $8 = 2^3$. Then the equation is, $\log_2(2^4) = (2^3)(2^{-x})$. Then, the left-hand side is equal to 4, and we get 2^{3-x} on the right-hand side using the properties of exponents [$(2^a) \times (2^b) = (2^{a+b})$]. Notice again that $4 = 2^2$. So we have $2^2 = 2^{3-x}$. Using the property of exponents that if $2^a = 2^b$ then $a = b$, we get $2 = (3 - x)$. Solving for x , get 1 as the answer.

19. The graph representing $|x - 4| \geq 2$ is

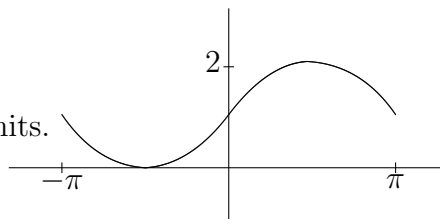


Answer:

Recall that $|x - 4|$ is the distance between x and 4. That distance is greater than 2 when x is more than 2 units to the left or right of 4; that is, $x < 2$ or $x > 6$.

20. Which of these choices best describes the alteration made to the graph of the sine curve, $y = \sin x$, for $-\pi \leq x \leq \pi$?

- (a) The amplitude of the graph was doubled.
 (b) The graph was shifted to the left π units.
 (c) The period of the graph was decreased to π units.
 (d) The graph was reflected about the x -axis.
 (e) The graph was shifted up 1 unit.



Answer:

For this question knowing two facts helps: First that the values of $\sin(x)$ lie between -1 and 1 and second, what the graph of $y = \sin x$ looks like. For the graph in the question, the values lie between 0 and 2 and the value at 0 is 1 while $\sin 0 = 0$ and at $\pi/2$ the value is 2 while $\sin \pi/2 = 1$. Therefore, the graph of sine has been shifted up by 1 . The process of elimination also helps to see that the other options do not work.

21. $\sin\left(\frac{3\pi}{4}\right) =$

- (a) $-\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$ (e) $\frac{\sqrt{3}}{2}$

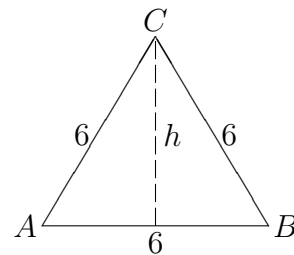
Answer:

For this question you need to know the idea of reference angles. The angle $\frac{3\pi}{4}$ is in the second quadrant, and its reference angle is $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$.

You also need to know the value of $\sin \frac{\pi}{4}$ which is $\frac{1}{\sqrt{2}}$. The values of sine in the second quadrant are positive, so the value of $\sin \frac{3\pi}{4}$ is also positive: $\frac{1}{\sqrt{2}}$.

22. Triangle ABC at the right is an equilateral triangle.
 The height h of the triangle is

- (a) 3 (b) $3\sqrt{3}$ (c) $6\sqrt{3}$ (d) $3\sqrt{2}$ (e) $6\sqrt{2}$



Answer:

For this question you need to know basic geometry and trigonometry. First know that all three angles in an equilateral triangle are equal to 60° . Then using trigonometry, we get $60^\circ = \frac{h}{6}$ (where h is the length of the perpendicular and 6 is the length of the hypotenuse of either right triangle). Then solve for h to get $3\sqrt{3}$. (You need to know that $60^\circ = \sqrt{\frac{3}{2}}$.)

23. When $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$, which of the following could possibly be $\tan \theta$?
(a) -8 (b) $-\frac{1}{8}$ (c) 0 (d) $\frac{1}{8}$ (e) 8

Answer:

For this question, knowing the graph of $\tan \theta$ helps: between $\frac{\pi}{2}$ and $\frac{3\pi}{4}$ we know from the graph that $\tan \theta$ is negative. Since $\tan \frac{3\pi}{4} = -1$, the answer has to be less than -1 which gives -8 as the answer.

24. For all real numbers x , $\cos^2 x - \sin^2 x =$
(a) 0 (b) 1 (c) $\sin(2x)$ (d) $\cos(2x)$ (e) $\cos(\frac{x}{2})$

Answer:

Here you need to know the double angle formula for cosine which is $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$. If $b = a$ the formula becomes $\cos(2a) = \cos^2(a) - \sin^2(a)$. Therefore, $\cos^2(x) - \sin^2(x)$ is equal to $\cos(2x)$.

25. If $f(x) = 10^{\frac{1+x}{1-x}}$, then $f(3) =$
(a) -100 (b) $-\frac{1}{100}$ (c) $\frac{1}{100}$ (d) 100 (e) 1000

Answer:

In this question, you need to evaluate at $x = 3$. The right side will look like: $10^{\frac{1+3}{1-3}}$. Simplify to get 10^{-2} which is equal to $\frac{1}{10^2}$. Therefore, the answer is $\frac{1}{100}$.

Answers

- | | | | | |
|------|-------|-------|-------|-------|
| 1. d | 6. b | 11. a | 16. a | 21. d |
| 2. d | 7. c | 12. a | 17. a | 22. b |
| 3. e | 8. e | 13. c | 18. e | 23. a |
| 4. a | 9. e | 14. b | 19. c | 24. d |
| 5. b | 10. a | 15. b | 20. e | 25. c |

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