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DO VAGUE PROBABILITIES REALLY SCOTCH PASCAL'S WAGER?

(Received in revised version 12 December 2002)

ABSTRACT. Alan Hájek has recently argued that certain assignments of vague probability defeat Pascal's Wager. In particular, he argues that "skeptical agnostics" – those whose probability for God's existence is vague over an interval containing zero – have nothing to fear from Pascal. In this paper, I make two arguments against Hájek: (1) that skeptical agnosticism is a form of dogmatism, and as such should be rejected; (2) that in any case, choice situations with vague probability assignments ought to be treated as "second-order" cases of choice under uncertainty, with the result that belief in God is the favored option in a very wide range of cases.

In his article (*Philosophical Studies*, Vol. 98, pp. 1–16),[1] Alan Hájek argued in novel fashion against Pascal's Wager: certain assignments of *vague probability* to God's existence, he claimed, "scotch the Wager."[2] I disagree. In this paper I will argue that defenders of the Wager have important replies that Hájek fails to explore.

I.

Let us begin with Hájek's target of criticism, namely, Pascal's argument as he describes it.[3] This consists of three premises:

> *Premise 1*. Rationality requires you to assign positive probability to God's existence.
> *Premise 2*. Either God exists or he does not exist, and you can either believe in God or not believe in God. The utilities of the relevant possible outcomes are as follows, where f_1, f_2, and f_3 are numbers whose values are not specified beyond the requirement that they be finite:

Philosophical Studies **112:** 279–290, 2003.

	God exists	God does not exist
Believe in God	∞	f_1
Do not believe in God	f_2	f_3

Premise 3. Rationality requires you to perform the action that maximizes expected utility.
Conclusion. Rationality requires you to believe in God.

This conclusion follows from the expected utility calculations, which are as follows (letting p be your positive probability for God's existence):

$$\text{EU(believe in God)} = \infty \cdot p + f_1(1 - p) = \infty$$
$$\text{EU(do not believe in God)} = f_2 \cdot p + f_3(1 - p) = \text{some finite value}$$

What goes wrong with this argument? According to Hájek, one thing wrong with it is premise 1. For starters, Hájek notes (following Oppy 1990) that premise 1 will be rejected by anyone who assigns probability 0 to God's existence – in other words, by anyone who is in Hájek's terms a *strict atheist*. Hájek does not press this point further against Pascal, however, for he doubts whether most professed non-believers are skeptical enough to qualify as strict atheists.[4] He does, though, think that many professed non-believers may well qualify as *skeptical agnostics*, that is, as people whose probability for God's existence is *vague* over an interval that includes 0.[5] What, though, does this mean?

Briefly, the idea is this. Most of the judgments we make about propositions do not involve assigning perfectly precise probability to those propositions. Rare is the person who judges there to be, say, a probability of 0.637 that he will receive the asking price for his house. Much more common is the person who judges he is, say, at least as likely as not to get the asking price. Decision theory should ideally find a way to accommodate this more common sort of person, and one obvious way is to permit interval-valued assignments of probability. That is to say, instead of assigning probability 0.637 to one's receiving the asking price for one's house, one simply assigns an upper and lower boundary for this probability. For example, a person who believes it as least as likely as not that he will receive the asking price would presumably give an interval assignment of [1/2,1] to the probability of this occurring.

We are now in a position to understand Hájek's skeptical agnostic: he or she assigns probability [0,x] to God's existence, where $0 < x \leq 1$. The term "skeptical" is meant to register the fact that this sort of agnosticism can never turn into belief in God. As Hájek notes,[6] this is so because Bayesian accounts of learning from experience (conditionalization, Jeffrey conditionalization) entail that assignments of zero probability are never to be updated to non-zero probability, no matter what new experiences one undergoes.[7] Hence any interval assignment that has zero as the lower boundary will *always* have zero as the lower boundary.

Hájek's next move is to argue that when the probability assigned to God's existence is vague over an interval containing zero it turns out that expected utility calculations no longer always tell in favor of belief in God. Before describing and evaluating this argument of Hájek's, however, I want here to pause and ask what follows if Hájek is right. Clearly he will have shown that skeptical agnostics can escape the Wager, just as strict atheists can. This result, however, will be of little significance if it turns out that no one ought to be a skeptical agnostic. And indeed I think this is the case. For consider again the fact that no sort of conceivable experience could get the skeptical agnostic to change her mind and become a believer. Now, I am not a believer myself, but I can conceive of some possible experiences that might get me to change this stance of mine. A booming voice from above followed by, say, a parting of a sea, witnessed by me and many independent others (including some with cameras) would do quite nicely. Or if that is deemed too crass, equally effective would be a booming voice from above followed by, say, a proof of Goldbach's Conjecture written in the sky. (Christian Goldbach conjectured in 1742 that every even integer greater than three is the sum of two primes.) Moreover, there are surely many experiences I might undergo *in an afterlife* that would work a similar conversion on me. And so on.

In short, I am unwilling to commit myself *never, ever* to believe in God, come what may. To do so would surely be pure dogmatism. Hence I suggest that the label "dogmatic agnostic" is a more fitting label for Hájek's non-believer than is "skeptical agnostic"; as such, it ought to be avoided by rational people. Hájek, though, appears not to agree, for he claims that rationality does in fact permit one to assign God's existence vague probability over an interval containing zero.[8] Only a scant amount of argument, however, backs up this claim. Hájek does in one passage say that such an assignment

> . . . could simply reflect the sentences to which you are prepared to assent (e.g., "the probability that God exists could be as low as 0, and could be as high as x . . ."), or even the bets you are prepared to enter into (e.g., somewhat fancifully, you refuse to pay anything for a bet that pays $1 iff God exists, and refuse to sell such a ticket for less than $x . . .).

All that this passage implies, however, is that some people can be accurately *described* as assigning God's existence some probability [0,x]. It in no way entails the *normative* claim that they would be rational to make such an assignment. With respect to Hájek's first comment regarding the sentences you are willing to assent to, I should hope that a rational aversion to dogmatism would lead you to withdraw your assent once you realize what such sentences entail. As for Hájek's second comment regarding the bets you are willing to enter into, it is worth noting that an assignment of [0,x] probability entails that you would refuse to pay *anything* for a bet on God's existence, *no matter what the bet pays*. But could it ever really be rational to refuse to spend even a *penny* for a bet that, say, pays $100 million if God exists?[9]

The upshot so far is that even if Hájek is right and people who assign probability [0,x] to God's existence can escape the wager, this will only be significant if such people can count as rational. Hájek has yet to show that they can.

II.

Suppose, though, that Hájek can through some further stretch of argument vindicate the rationality of "skeptical agnostics." In that case his argument that such people are immune to Pascal's Wager will deserve attention. Let us, then, examine this argument.

The argument begins by calculating expected utilities using the vague probability interval [0,x] for God's existence. This is done "point-by-point," that is, by comparing the expected utilities that result from letting the probability of God's existence take on each value in the interval [0,x]. Doing this, it turns out that precisifying with zero probability leads to an expectation for belief in God of

$$\infty \cdot 0 + f_1(1 - 0) = f_1$$

while precifying with each non-zero probability ($0 < p \leq x$) leads to an expectation of

$$\infty \cdot p + f_1(1 - p) = \infty.$$

Hence the expectation for belief in God is vague over the two-member set $\{f_1, \infty\}$. With regard to non-belief, the same method reveals that this option has an expectation that is vague over the interval

$$[f_2 \cdot 0 + f_3(1 - 0), f_2 \cdot x + f_3(1 - x)] = [f_3, f_2 \cdot x + f_3(1 - x)]$$

Having computed these expectation sets, Hájek goes on to note that, contrary to Pascal's argument, the rational thing to do now depends on the values of f_1 and f_3. If $f_1 \geq f_3$, then Pascal gets his result: the rational thing to do is believe in God, for believers are at least as well off as non-believers whether God exists or not, and they are better off if God exists. But if $f_1 < f_3$, says Hájek, "[t]hen there is no univocal answer. Precisfying with non-zero probabilities, belief is rationally required; precisifying with zero probability, non-belief is rationally required. Thus it is *not* the case that rationality requires you to believe in God."[10]

I believe the Pascalian wagerer has an effective reply to this charge of Hájek's. As a lead-in to this reply, let us consider Hájek's discussion of another approach to the choice at hand. In a lengthy footnote Hájek takes up a suggestion from Michael Thau that one might compare the expectation sets not with a point-by-point approach (that is, not by dividing the expectation sets into those produced by zero probabilities and those produced by non-zero probabilities), but rather with a "global" approach.[11] That is to say, one might simply ask, "Given that the expectation for belief ranges over $\{f_1, \infty\}$, while the expectation for non-belief ranges over $[f_3, f_2 \cdot x + f_3(1 - x)]$, which option should I choose?"

Hájek proposes the following rule for such a global comparison: "If action A_1 has vague expectation set S_1 and action A_2 has vague expectation set S_2, then A_1 is determinately better than A_2 iff each point in S_1 is greater than every point in S_2; A_2 is determinately better than A_1 iff each point in S_2 is greater than every point in S_1; otherwise it is indeterminate which action is better."[12] This rule, however, seems to me unacceptable. For instance, suppose you are offered, free of charge, a choice between two lottery tickets: Lottery A pays $3000; the odds of winning are known only to be between 0 and 2/3. Lottery B pays $5997; the odds of winning are known only to be between 1/3 and 1. Which ticket should you choose? Surely the rational choice here is lottery B. And yet Hájek's rule implies that it is indeterminate which lottery is rationally preferable. To see this, compare their expectation sets: Lottery A has an expectation interval of [$0, $2000] while B has an expectation interval of [$1999, $5997]. Because they overlap, the choice between them is indeterminate according to Hájek's rule.

I suggest that a more rational approach is to treat this choice as a "second-order" choice under uncertainty. By this I mean that you should treat the case, not as a straightforward case of uncertainty in which you are unable to assign any probabilities to possible outcomes (it is not that since you can assign *vague* probabilities), but rather as a case in which you are unable to assign any "second-order" probability to claims that the first-order probabilities will take on such-and-such a value. This is a natural suggestion, for it is natural to suppose that in assigning vague probability to a proposition one is in the usual case saying "With this interval assignment I have reached the limits of my knowledge regarding probabilities; beyond this assignment I am *uncertain* what more to say."[13]

If one thinks of a choice involving vague probabilities as a second-order choice under uncertainty, then one can approach the choice by letting the familiar principles of rational choice under uncertainty likewise take on a second-order form.[14] The maximin principle, for instance, will instruct you to choose the option with the highest minimum possible *expectation* (as opposed to the highest minimum possible utility), namely, lottery B. The minimax regret principle will instruct you to minimize the maximal "expectation regret" you might feel should the lottery you pick have in fact a lower expectation than the other lottery. Inasmuch as the maximal expectation regret for lottery A is \$1999 while the corresponding regret for lottery B is \$1, this principle clearly favors lottery B. The optimism-pessimisim principle will instruct you, for each option, to take a weighted average of that option's minimum and maximum possible expectations, and then choose the option with the highest such average. Since lottery B has both the highest possible minimum and maximum, this principle will rank it first.[15]

III.

What happens when we apply this approach to the choice between belief and non-belief in a case where vague probability [0,x] is assigned to God's existence? Recall that this probability assignment generates the expectation set $\{f_1, \infty\}$ for belief and the expectation set $[f_3, f_2 \cdot x + f_3(1 - x)]$ for non-belief. Which of these options will be favored by the second-order forms of the familiar principles for choice under certainty? It turns out that both the minimax regret principle and the optimism-pessimism principle will favor belief, the first because non-belief has the highest possible expectation regret (∞), the second because the weighted optimism-pessimism average for belief will itself equal ∞, since ∞ is one of the elements in the average. If either of these is a trustworthy principle to follow in this case, then belief is the superior option, and Hájek is wrong to think that vague probabilities scotch the Wager.[16]

Things are not so simple as regards the second order form of the maximin principle, however, for here the relative sizes of the variables f_1, f_2, and f_3 do matter. The maximin principle will favor non-belief over belief if and only if the minimum of the expectation interval $[f_3, f_2 \cdot x + f_3(1 - x)]$ is greater than f_1. In order to determine which is greater we need to know something about the relative sizes of the f's, just as Hájek has claimed. With respect to this principle at least, it seems that Hájek has indeed scored some points against the Wager.

Things, though, may not be as simple as they seem. For defenders of Pascal can plausibly question the appropriateness of the maximin principle as regards the current choice situation. It is well known that in many situations this principle delivers counter-intuitive results.

Suppose for instance you know only that lottery A pays \$100 if a red ball is drawn and \$200 otherwise, while lottery B pays \$99 if a red ball is drawn and \$10,000 otherwise. In this case the maximin principle will instruct you to choose lottery A. For this reason philosophers have sought to define more narrowly the situations in which it is reasonable to rely on the maximin principle. One influential discussion in this regard is that of John Rawls in his *A Theory of Justice*.[17] Here Rawls lists three conditions that make it reasonable to apply the maximin principle: (1) probabilities cannot be known, or are very insecure (that is, this is a choice under uncertainty in the decision-theoretic sense); (2) the worst-case scenario for the chosen option is still a very satisfactory outcome; (3) the rejected options have worst-case scenarios that are intolerable.

These conditions open the door for defenders of Pascal to reply that the second-order form of the maximin principle should not be used to choose non-belief. For instance, they can point out that condition (3) will only be met so long as the worst-case expectation of the rejected alternative (belief) – namely, f_1 (the well-being of believers in a godless universe) – is intolerably low. But this does not seem on average to be the case. Moreover, and more importantly, defenders of Pascal can insist that the minimum possible expectation associated with *non*-belief is not at all guaranteed to be satisfactory, and hence there is no guarantee that condition (2) is met.

This latter point is worth exploring in some detail. To begin with, note that by listing the expectation interval associated with non-belief as $[f_3, f_2 \cdot x + f_3(1 - x)]$, Hájek gives the appearance that f_3 is the lowest bound for this interval. This need not be, however; in fact f_3 will be the *upper* bound whenever $f_2 < f_3$ – whenever, that is, God makes non-believers worse-off than they would be in a godless universe. It is not difficult to imagine a god who does this; in the literature on Pascal's Wager, for example, it is at least as common as not to suppose that non-believers are punished in hell, with well-being in hell being represented as $-\infty$. It is true that Pascal himself does not emphasize the prospect of damnation in formulating his wager,[18] but inasmuch as this prospect is certainly relevant to the choice Pascal presents, incorporating it into the Wager seems a natural extension of Pascal's argument.

What follows in this case? Letting $f_2 = -\infty$, we can compute the expectation for non-belief as follows. Precisification with zero probability leads to an expectation for belief in God of

$$-\infty \cdot 0 + f_3(1 - 0) = f_3$$

while precification with each of the non-zero probabilities ($0 < p \leq x$) leads to an expectation of

$$-\infty \cdot p + f_3(1 - p) = -\infty.$$

Hence with the inclusion of hell, the expectation for non-belief is vague over the two member set $\{-\infty, f_3\}$. In this case the maximin principle (as well as the other principles) resoundingly favors belief. With respect to *this* formulation of wager, then, vague probabilities are no help; the non-believer must assign precisely zero probability to God's existence. Only strict atheists are off the hook.

What, though, about a case in which $f_2 < f_3$, but only *finitely* so? Recall that in this case the maximin principle tells us that non-belief is rationally permissible if and only if the quantity $f_2 \cdot x + f_3(1 - x)$ (the minimum possible expectation of non-belief) is at least as great as f_1 (the minimum possible expectation of belief). When will this be? To answer this question, we can note the following:

$$\begin{aligned} f_2 \cdot x + f_3(1 - x) \geq f_1 &\quad \text{if and only if} \quad f_2 \cdot x + f_3 - f_3 \cdot x \geq f_1 \\ \text{"} &\quad \text{if and only if} \quad x(f_2 - f_3) \geq f_1 - f_3 \\ \text{"} &\quad \text{if and only if} \quad x \leq \frac{f_1 - f_3}{f_2 - f_3} \end{aligned}$$

(The inequality in the last line is reversed since $f_2 - f_3 < 0$.) This is significant, for the quantity $f_1 - f_3$ represents the cost of false religious belief while the quantity $f_2 - f_3$ represents the cost of divine punishment for non-belief. As a result we can say that the maximin principle will reject non-belief in any version of the Wager in which the upper bound of the probability for God is greater than the ratio between the cost of false belief and the cost of punishment for non-belief.

Some examples will make this more intuitive. For instance, if the cost of punishment is twice as great as the cost of false belief, then one must assign probability [0,1/2] or less to God's existence before the maximin principle will permit non-belief. If the cost of punishment is 1000 times as great as the cost of false belief, then one must assign probability [0,1/1000] or less to God's existence before the maximin principle will permit non-belief. And so on. This makes sense; it is merely states more precisely a very commonsensical idea, namely, that the more severe divine punishment is, the more confident the non-believer must be in his or her rejection of belief.

In short, if I am right that choice situations with vague probability assignments should be approached as second-order choices under uncertainty, then defenders of Pascal can claim victory against any given skeptical agnostic (1) by rejecting strict atheism and skeptical agnosticism as forms of dogmatism; OR (2) by using either the minimax regret principle or the optimism-pessimism principle to choose under uncertainty; OR (3) by using the maximin principle and reckoning the cost of divine punishment to be so large that the cost ratio of false belief to punishment is less than the upper probability bound that the skeptical agnostic assigns to God's existence.

In short, Pascal's defenders have several ways of escaping Hájek's challenge. I conclude, then, that vague probabilities do not "scotch" Pascal's Wager; for many, many versions of the Wager, vague probabilities lead instead to a "Scotch verdict" of "not proven guilty" – or more to the point, "not proven unsound."[19,20]

NOTES

[1] Hájek (2000).
[2] Ibid., p. 1.
[3] Ibid., pp. 1–2.
[4] Ibid., p. 3.
[5] As Hájek notes, Van Fraassen has argued that *all* agnostics should be represented as assigning this sort of probability to God's existence (Van Fraassen, 1989). Hájek convincingly disputes this claim of Van Fraassen's in Hájek (1998).
[6] Hájek (2000, pp. 7 and 13, note 5).
[7] For a brief explanation of this, see Howson (2000, p. 70).
[8] Hájek (2000, p. 7).
[9] Hájek also suggests that choice under uncertainty (in the decision-theoretic sense) can be understood as choice under risk with totally vague probability [0,1] (p. 14, n. 6). However, I doubt whether uncertainty is best thought of as totally

vague probability. For were one to assign probability [0,1] to some proposition and then update one's probabilities via some orthodox sort of conditionalization, one would never be able change one's assignment of totally vague probability, no matter what one came to experience. (Assignments of 1, like assignments of 0, cannot be changed via conditionalization.) We might say that someone who assigns probability [0,1] to some proposition p is *rigidly* uncertain of p – that is, uncertain of p in all possible worlds. I doubt, though, whether anyone is ever genuinely rigidly uncertain.

[10] Ibid., p. 8.

[11] Ibid., p. 14, note 7.

[12] Ibid.

[13] This is not to deny that in some unusual cases second-order probability assignments will be possible.

[14] For a classic discussion of choice under uncertainty, see Luce and Raiffa (1957), Chapter Thirteen.

[15] It is less clear how to apply the notorious "indifference principle." This principle (in its first-order form) says that absent any other knowledge you should treat all possible outcomes as equiprobable. It is less clear how to apply a second-order version of this principle because there are infinitely many possible values the probabilities can take on within the probability intervals. The following line of thought seems to me to indicate the correct method, however. Consider a lottery with vague probability [a,b] that pays a cash prize of C. Choosing *two* points in this interval (the bounds a and b) and applying the indifference principle yields an "expected expectation" of $\frac{a+b}{2} \cdot C$. Choosing *three* points in this interval (the bounds and midpoint) yields an expected expectation of $\frac{1}{3}a \cdot C + \frac{1}{3}\left[a + \frac{b-a}{2}\right] \cdot C + \frac{1}{3}b \cdot C$, which after simplification equals $\frac{a+b}{2} \cdot C$. In general, choosing n points in this interval yields an expected expectation of

$$C \cdot \sum_{i=0}^{n-1} \frac{1}{n}\left[a + (b-a) \cdot \frac{i}{n-1}\right] =$$

$$C \cdot \left[a + (b-a)\sum_{i=0}^{n-1} \frac{i}{n(n-1)}\right] = C\left[a + \frac{b-a}{2}\right] = \frac{a+b}{2} \cdot C$$

Clearly the limit of this as n approaches infinity is equal to $\frac{a+b}{2} \cdot C$. Computing things in this way yields an expected expectation of \$1000 for lottery A and \$3998 for lottery B.

[16] This same is true regarding the indifference principle if it is applied in the manner proposed in the previous footnote, for then the expected expectation of belief = ∞.

[17] Rawls (1999, p. 134).

[18] See Pascal (1966 [1670]), fragment 418, "Infini-rien."

[19] I do, though, think a version of the *Many Gods Objection* can prove the Wager "guilty," namely, one that insists that God (should he exist) is at least as likely

to punish *believers* and reward *non*-believers as he is to do the reverse. (I say "at least as likely," for I do not think one can defeat the Wager merely by insisting that there is *some* positive probability that such a believer-hating God exists. I explain my reasons for thinking this in Duncan (2002).) More simply, I believe that one can also defeat the Wager by insisting that we are completely ignorant as to how God (should he exist) treats believers relative to non-believers in the next life (should this exist).

[20] I would like to thank the anonymous reviewer from *Philosophical Studies* whose helpful comments on a draft of this paper led to significant improvements.

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