

2023 Math Exploration Day

Team Competition

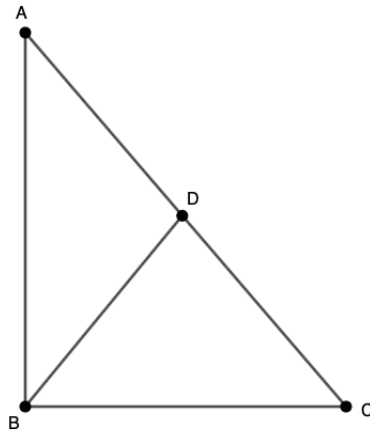
1. Twenty-five percent less than 60 is two-thirds more than what integer?

2. You drop a bouncy ball from 10-meter platform and each time it bounces back up exactly two-thirds of its previous height. What is the total vertical distance traveled by the ball when it reaches its highest point after the third bounce? Round to the nearest tenth of a meter.

3. Compute the reciprocal of the sum of the reciprocals of the two smallest 2-digit prime numbers. Express your answer in the form $\frac{a}{b}$, where a and b are whole numbers.

4. Compute the sum of the roots of the equation $(2x + 4)(x - 2) + (x + 2)(x - 5) = 0$.

5. In right triangle ABC , the measure of angle BAC is 38° . The length of segment BD equals the length of segment CD . What is the measure of the angle BDA ?



6. Given $f(x) = 1 - \frac{1}{x}$, compute $f(f(f(2023)))$. Provide your answer in simplified form.

7. Define the operation \star by $x \star y = x^2 + xy - y^2$. What is the value of $(2 \star 3) \star (4 \star 5)$?

8. A coin is flipped 4 times. What is the probability that at least 2 heads are obtained?

9. In a quadrilateral $ABCD$, we have:

- the measure of $\angle B = 3 \times$ (the measure of $\angle A$),
- the measure of $\angle C = 4 \times$ (the measure of $\angle B$), and
- the measure of $\angle D = 2 \times$ (the measure of $\angle C$).

What is the measure of $\angle A$?

10. You hiked up a hill at rate of 1.5 miles per hour and came down at a rate of 4.5 miles per hour. It took you a total of 6 hours to make the hike up to the top and back down. How many miles was it to the top of the hill?

11. If you add the square of my dog's age to my cat's age, you get 14. If you add the square of my cat's age to my dog's age, you get 28. How old is my cat? Both ages are given in whole numbers of years.

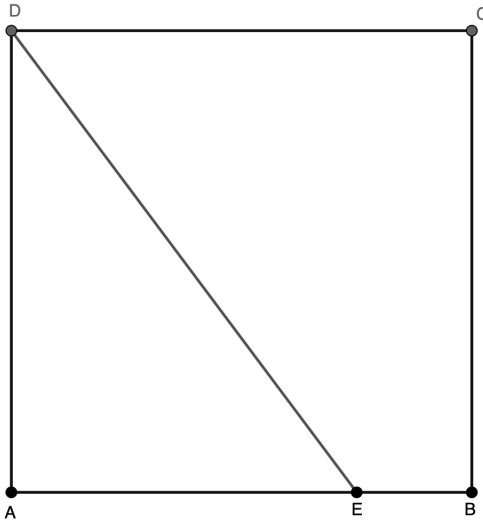
12. Aaron's car averages 30 miles per gallon of gasoline and Osman's truck averages 15 miles per gallon of gasoline. If they both drive the same number of miles, what is their cars' combined rate of miles per gallon of gasoline?

13. Use the digits 1, 2, 3, 4, 6, 7, 8, and 9, to form 4 two-digit prime numbers, using each digit only once. What is the sum of the 4 prime numbers?

14. You are building a scale model of your town's water tower. It is 50 meters to the top of the town's tower. The top portion of the tower is a sphere that holds 100,000 liters of water. Your model's sphere holds 0.1 liters. How many meters tall should you make your tower?

15. The sum of two positive numbers is 6 times their difference. What is the ratio of the larger number to the smaller number?

16. The image shows a square field that has area of 40 acres. The line DE is a straight path and E is 110 yards away from B . In a race, Teresa runs directly from A to B , but Megan runs from D to E and then to B . Megan and Teresa begin the race at the same time. Each runs at their own constant speed and when Megan reaches E , Teresa is 30 yards ahead of her. Who wins the race, and by how much? (There are 4840 square yards in an acre.)



17. Find two positive integers whose product is 6930 and which have the smallest possible difference.

18. Solve for y : $\ln(\ln(729)) - \ln y = \ln(27)$

19. Starting with 3, what is the sum of the first 2023 whole numbers that are 3 apart?
That is, what is $3 + 6 + 9 + \dots$, ending with the 2023rd multiple of 3?

20. A circle of radius 2 sits tangent (just touching) the x - and y -axes. The length of line segment \overline{CD} equals the length of line segment \overline{DB} . What is the area of triangle ABC ?

