2025 IC Math Exploration Day
Team Competition

1. Compute $\frac{2+4+6+8}{1+3+5+7} - \frac{1+3+5+7}{2+4+6+8}$, expressing the answer in the form $\frac{a}{b}$ written in lowest terms.

2. The cube of a positive integer is 49 times that integer. What is that integer?

3. If $\sec(A) = \frac{13}{5}$, then what is the value of $\left|\tan(A)\right|$? Express your answer in the form $\frac{a}{b}$.

4. Define the binary operation ☺ by $x☺y = x^2 \cdot y + y^2 \cdot x$. Compute $1☺(2☺3)$.

5. Find the sum of all solutions of $\left|x+1\right| + \left||x|-2\right| = 5$.

6. The geometric mean of three numbers (say, a, b, c) is $\sqrt[3]{a \cdot b \cdot c}$. If the geometric mean of $3, 15$, and c is 45, then what is the value of c?

7. The vertices of triangle ABC lie on the parabola $y = 4x^2$ with A at $(0, 0)$ and the side $\overline{BC}$ perpendicular to the y-axis. If the area of the triangle is 500, what is the length of $\overline{BC}$?

8. A bag has 6 M&Ms remaining: 2 green, 2 yellow, and 2 red. Armin takes two at random, then Beth takes two at random, and Carlo is left with the final two. What is the probability that Carlo gets two M&Ms of the same color?

9. The average grade in my morning Algebra I class of 30 students is 81. The average grade in my afternoon Algebra I class of 24 students is 90. What is the average grade of all students in these morning and afternoon classes?

10. In their latest game, the IC women's basketball team was successful on 60% of their 2-point shots and 30% of their 3-point shots. This yielded 66 points. If they attempted twice as many 2-point shots as 3-point shots, how many successful 3-point shots did they make?

11. Each student in a geometry class are asked to draw a circle of radius 2 and a circle of radius 4 on a page in their notebook. Then, they are asked to draw all possible lines that are simultaneously tangent to both circles and record the number of lines they see. How many different correct answers are possible?

12. Gabbie and Michael play a game in which they each flip a fair coin. Each one flips their coin until it lands heads up and then they stop. What is the probability that both of them flip their coins the same number of times?

13. The sum of two positive integers is six times their difference. What is the ratio of the smaller number to the larger?

14. The interval of solutions of the inequality $m \leq 3x + 5 \leq n$ has length 15. What is $n - m$?

15. How many perfect squares are there that are less than or equal to 2025 and divisible by 3?

16. Consider the sequence $a_1 = 1$ and for $n > 1$, we have $a_n = 1 + \frac{1}{a_{n-1}}$. What is the value of a_{10}? Express your answer in the form $\frac{a}{b}$.

17. The imaginary number, i, is defined by $i^2 = -1$. What is the value of the following sum:

$$\sum_{k=0}^{2025} i^k = 1 + i + i^2 + i^3 + i^4 + i^5 + \cdots + i^{2024} + i^{2025}?$$

18. What is the value of x for $\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_5 x} = 1$?

19. The numbers 3, 4, 5, 6, 7, and 8 are placed one each on the faces of a six-sided die. For each of the eight vertices of the die, compute the product of the numbers assigned to the three faces that contain that vertex. What is the largest possible sum of the eight products?

20. $\sigma(n)$ = the sum of all positive factors of n. For example, $\sigma(12) = 28$ because the positive factors of 12 are 1, 2, 3, 4, 6, and 12. Compute $\sigma(2025)$.