2026 IC Math Exploration Day
Team Competition - Answers

1. $293,335$
2. 640
3. 9
4. 7
5. 3
6. 0.0059
7. 40
8. 61
9. 2116
10. 3:00pm
11. 49
12. 47
13. -1
14. $1,000,000$
15. 4
16. 80
17. 8
18. 6:09:45pm, or 45 seconds after 6:09pm
19. 2
20. 51/13

2026 IC Math Exploration Day
Team Competition - Solutions

1. What is the sum of every multiple of 7 from 1 to 2026?

   $\boxed{293{,}335}$

   There are 289 multiples of 7 from 1 to 2026. So, the sum of all 289 multiples is

   $$7 \cdot (1 + 2 + 3 + \cdots + 289) = 7 \cdot \frac{289 \cdot 290}{2} = 293,335.$$

2. The Giant Pacific Octopus has (like most octopi) eight arms with large suction cups arranged in two rows along each arms. Each row contains 100 - 140 suction cups. What is the difference between the maximum total number of suction cups and minimum total number of suction cups on such an octopus?

   $\boxed{640}$

   The difference is

   $$140 \cdot 2 \cdot 8 - 100 \cdot 2 \cdot 8 = 40 \cdot 16 = 640.$$

3. Start with $n = 10^{100}$ and compute its square root, then compute the square root of the result, then compute the square root of that result, and so on. What is the minimum number of times that you must repeatedly apply the square root to $n = 10^{100}$ in this way so that the result is less than 2?

   $\boxed{9}$

   Let k be the number of times we need to apply the square root to n so that the result is less than 2. This means that $n^{1/2^k} < 2$. Let's use natural logarithm (though any logarithm will work) to solve this.

   $$\ln\left(n^{1/2^k}\right) < \ln(2) \Longrightarrow \frac{1}{2^k}\ln(n) < \ln(2) \Longrightarrow \ln(n) < 2^k \ln(2)$$

   Applying logarithm again,

   $$\begin{aligned}
   \ln\left(2^k \ln(2)\right) > \ln(\ln(n)) &\Longrightarrow k\ln(2) + \ln(\ln(2)) > \ln(\ln(n)) \\
   &\Longrightarrow k > \frac{\ln(\ln(n)) - \ln(\ln(2))}{\ln(2)} \\
   &\Longrightarrow k > \frac{\ln\left(\ln\left(10^{100}\right)\right) - \ln(\ln(2))}{\ln(2)} \\
   &\Longrightarrow k > \frac{\ln\left(100\ln(10)\right) - \ln(\ln(2))}{\ln(2)} \\
   &\Longrightarrow k > \frac{\ln(100) + \ln((\ln(10)) - \ln(\ln(2))}{\ln(2)} > 8.3
   \end{aligned}$$

   So, the minimum is 9.

4. Define the binary operation $*$ by $x * y = x^2 - 2y^2 + xy$. If $x * y = 22$ and $x = 8$ and y is positive, then what is the numerical value of y?

   $\boxed{7}$

   $$22 = x * y = (x + 2y)(x - y) = (8 + 2y)(8 - y) \Longrightarrow y = 7.$$

5. A bag contains 12 marbles, some of which are blue. If 6 blue marbles are added to the bag, the probability of drawing a blue marble is double what it was before. How many blue marbles were originally in the bag?

   $\boxed{3}$

   Let b be the number of blue marbles originally in the bag. Then, comparing the probabilities, we have

   $$\frac{b+6}{12+6} = 2\frac{b}{12} \Longrightarrow b + 6 = 3b \Longrightarrow 2b = 6 \Longrightarrow b = 3.$$

6. The minute hand on Cornell University's McGraw Tower clock is five feet long. Find the average speed of the tip of the minute hand in miles per hour. Give your answer to 4 decimal places.

   $\boxed{0.0059}$

   The tip of the minute hand travels $2\pi \cdot 5$ feet every hour. So, the speed in miles per hour is

$$2\pi \cdot 5 \cdot \frac{1}{5280} = 0.005949986$$

7. A large bin contains 100 pounds of a dog food mix that is 50 percent beef, 30 percent chicken, and 20 percent salmon. A second mix of dog food containing 20 percent beef, 40 percent chicken, and 40 percent salmon is added to the bin. The combined mix contains 40 percent beef. How many pounds of salmon are now in the bin?

   $\boxed{40}$

   Let x be the number of pounds of the second mix added to the bin. There are 50 pounds of beef originally in the bin. So,

$$0.40 \cdot (100 + x) = 50 + 0.20x \Longrightarrow 0.20x = 10 \Longrightarrow x = 50.$$

   So, 50 pounds of the second mix are added to the bin, with 40% of that mix being salmon, meaning that 20 pounds of salmon are added to the bin with the second mix. Originally, there are 20 pounds of salmon in the bin, so now there is a total of 40 pounds of salmon in the bin.

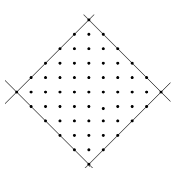
8. How many pairs of integers (x, y) satisfy the following system of inequalities?

$$\begin{aligned} x + y &\leq 20 \\ y - x &\leq 10 \\ x + y &\geq 10 \\ y &\geq x \end{aligned}$$

   $\boxed{61}$

   The region defined by this set of inequalities with the points with integer coordinates is shown below. Working from left to right vertically in this region, we see that the number of points in the region with integer coordinates is:

$$1 + 3 + 5 + 7 + 9 + 11 + 9 + 7 + 5 + 3 + 1 = 61.$$



9. You planted an orchard of apple trees in rows to form a square, and when you were done, you had 148 trees left over. You purchased 32 more trees and planted these and the previously leftover ones to expand the square by one more row in each direction. How many trees were planted when you were done?

   $\boxed{2116}$

   Let n be the number of trees in each row in the original square. Then,

$$(n + 2)^2 = n^2 + 148 + 32 \Longrightarrow 4n + 4 = 180 \Longrightarrow n = \frac{176}{4} = 44.$$

   In the end, you planted $46^2 = 44^2 + 148 + 32 = 2116$ trees.

10. You leave your house at 12:30pm on your bicycle traveling due west at a constant speed of 6 miles per hour. Your sister leaves your house at 1:30pm on her bicycle traveling due south at 10 miles per hour. At what time are you both exactly the same distance from your house?

$\boxed{\text{3:00pm}}$

Let t be the time, in hours, since 12:30pm. Then, when the two are the same distance from the house, we have that the number of hours taken to reach this is

$$6t = 10(t-1) \Longrightarrow 4t = 10 \Longrightarrow t = 2.5$$

So, it will be 3:00pm when both are the same distance from the house.

11. The area of a right triangle is 330 and its hypotenuse has length 61. What is the absolute value of the difference between the two non-hypotenuse sides of the triangle?

$\boxed{49}$

Let a and b be the sides of the triangle, and assume $a \geq b$. We have $a \cdot b = 660$ and $a^2 + b^2 = 61^2 = 3721$. Then,

$$(a-b)^2 = a^2 - 2ab + b^2 = 3721 - 1320 = 2401.$$

So, $a - b = 49$.

12. The sum of the cube of a number x and the reciprocal of its cube is 7. Find the sum of x^6 and its reciprocal.

$\boxed{47}$

$$x^6 + \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right)^2 - 2 = 7^2 - 2 = 47.$$

13. Consider the follow set of numbers $S = \{3, 4, 7, 4, 1, 6, 8, 13\}$. You create a new set T by adding an additional number to S. What number do you need to add so that the average of T is equal to the median of S?

$\boxed{-1}$

The median of S is 5. Let x be the number added to S to form T. Then, the average of T is

$$\frac{3+4+7+4+1+6+8+13+x}{9} = \frac{46+x}{9}.$$

Setting the average equal to the median of S yields

$$\frac{46+x}{9} = 5 \Longrightarrow 46 + x = 45 \Longrightarrow x = -1.$$

14. What is the value of $\dfrac{1001! - 1000!}{999!}$?

$\boxed{1,000,000}$

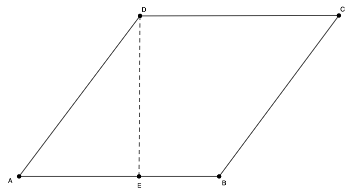
$$\frac{1001! - 1000!}{999!} = \frac{1000!(1001-1)}{999!} = (1000)(1000) = 1,000,000.$$

15. For what value of t does $3^t \cdot 9^{4t} = 81^9$?

$\boxed{4}$

$$3^t \cdot 9^{4t} = 81^9 \Longrightarrow 3^t \cdot 3^{8t} = 3^{4 \cdot 9} \Longrightarrow 3^{9t} = 3^{36} \Longrightarrow 9t = 36 \Longrightarrow t = 4.$$

16. In the rhombus $ABCD$, we have $\overline{DE} \perp \overline{AB}$ and $\overline{EB}$ has length 4 and is 40% of the length of $\overline{AB}$. What is the area of the rhombus?



$\boxed{80}$

Since $\overline{EB}$ has length 4 and is 40% of $\overline{AB}$, we see that $\overline{AB}$ has length 10 and $\overline{AE}$ has length 6. Because this is a rhombus, $\overline{AD}$ has length 10. Using the Pythagorean Theorem, $\overline{DE}$ has length 8. Thus, the area of the rhombus is $10 \cdot 8 = 80$.

17. For how many values of b does the polynomial $x^2 + bx + 100$ have two distinct integer roots?

$\boxed{8}$

To have two distinct integer roots (r_1 and r_2), the polynomial must factor as $(x - r_1)(x - r_2)$ with $r_1 r_2 = 100$ and $b = -(r_1 + r_2)$. We can enumerate the possibilities.

r_1	r_2	b
1	100	-101
-1	-100	101
2	50	-52
-2	-50	52
4	25	-29
-4	-25	29
5	20	-25
-5	-20	25

In each r_1, r_2 paring above, we can swap that the order but this results in the same b value. So, there are 8 values of b that work.

18. You begin an epic sneezing fit at 2:00pm one day, sneezing then and every 15 seconds thereafter. At what time do you have your 1000^{th} sneeze?

$\boxed{\text{6:09:45pm, or 45 seconds after 6:09pm}}$

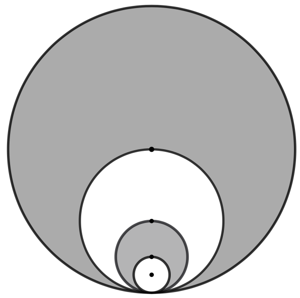
The 1000^{th} sneeze will be the 999^{th} sneeze after 2:00pm, and this will occur at $999 \cdot 15 = 14985$ seconds. This is the same as 249 minutes and 45 seconds, or 4 hours 9 minutes 45 seconds. So, the 1000^{th} sneeze occurs at 6:09:45pm, or 45 seconds after 6:09pm.

19. Let $z = 3 + i$. If we draw, in the complex plane, the line determined by z and z^2, what purely real number, x, is on this line?

$\boxed{2}$

$z^2 = (3 + i)^2 = 8 + 6i$ and the line determined by z and z^2 is the line between points $(3, 1)$ and $(8, 6)$ in the plane. This line has equation $y - 1 = x - 3$ and we want the x-intercept. Setting $y = 0$, yields $x = 2$.

20. Four circles are tangent to each other, as in the figure below. The centers of the circles are shown. Compute the ratio of the shaded area to the non-shaded area. Express your answer in the form $\frac{a}{b}$, with a and b whole numbers.



$\boxed{51/13}$

Let r be the radius of the smallest circle. Then, in increasing order, the radii of the three larger circles are $2r, 4r$, and $8r$. The shaded area is

$$\pi \cdot 64r^2 - \pi \cdot 16r^2 + \pi \cdot 4r^2 - \pi \cdot r^2 = 51\pi r^2.$$

The non-shaded area is

$$\pi \cdot 16r^2 - \pi \cdot 4r^2 + \pi \cdot r^2 = 13\pi r^2.$$

Thus, ratio of the shaded area to the non-shaded area is $\frac{51}{13}$.