Problem Set 4

1. [10 points] The diameter of the nucleus is about 10^{-14} m wide. Suppose you have a neutron confined to a one-dimensional box of length 10^{-14} m. Obtain an expression of the energy levels (in J) of the neutron. Sketch the locations of the first five energy levels on a vertical axis. Discuss your result.

2. [10 points] Use the uncertainty principle to compare the velocities of an electron and a proton confined to a box of width 1 nm.

3. [10 points] We have discussed the fact that a hydrogen atom is about 5.3×10^{-11} m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom. Compare this with the known energy of an electron in a hydrogen atom, which is 13.6 eV, or 2.2×10^{-18} J.

4. [20 points] In class we stated that in a hydrogen atom, the wavelength of the electron has to be related to the classical radius of the electron's orbit via $\lambda = 2\pi r/n$, where n was any integer. The argument was that if $n\lambda$ was not an integer multiple of $2\pi r$, then after several times around the hydrogen atom, you would get destructive interference, hence no electron!

   Let's assume we have a wavelength of 2π (arbitrary length units) and the circumference of our orbit is $2\pi - \pi/5$. Now we have the situation described in class: the wavelength is a little bit longer than out orbit: just a little bit less than one wavelength fits onto one orbit.

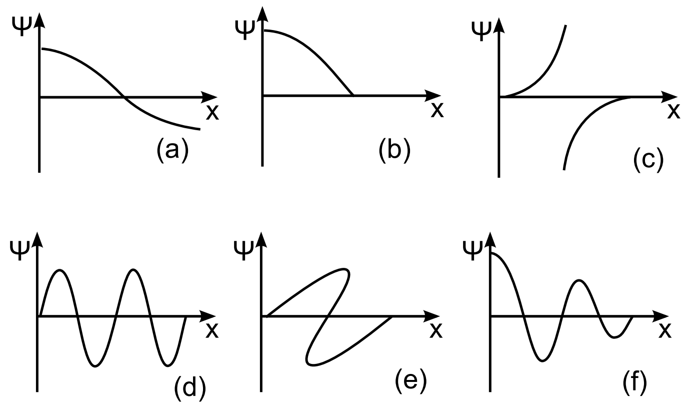
   Now let's use a plotting program like Excel, Matlab, or Mathematica to see the effects of destructive interference. Plot $\sin(x)$ for x from $x = 0$ to $x = 2\pi - \pi/5$. This will represent the electron's first time around the circle. Notice that a little bit less than one wavelength fits on one orbit.

   However, if the wavelength doesn't fit on the orbit, then by the time the electron returns to the where it started, we have a sine function starting with a different phase. Plot $\sin(x - \pi/5)$ from 0 to $2\pi - \pi/5$ to represent the second time around the circle. Convince yourself that this looks like the second time around the circle, and explain why this is true. Then, add the second time around to the first time around to see the net effect. Is there more or less "waviness"?

   Plot $\sin(x - 2\pi/5)$ from 0 to 2π to represent the third time around the circle, and then Plot $\sin(x - 3\pi/5)$ for the fourth time, etc. After ten times around the circle, add up the net effect. Discuss your results.

5. [10 points] Suppose we have an electron in orbit around the proton in hydrogen, and the radius of this orbit is 1 mm. What is the energy of the electron in this state? What energy level is the electron in? How likely is is it for the electron to remain in this orbit?

6. [10 points] Of the wave functions shown below, which cannot have physical significance? Why not?



7. [15 points] Suppose there is a particle limited to the x axis and has the wave function $\psi = ax$ between $x = 0$ and $x = 1$; everywhere else $\psi = 0$.
   (a) Normalize this wave function.
   (b) Find the probability that the particle can be found between $x = 0.45$ and $x = 0.55$.
   (c) Find the expectation value for $\langle x \rangle$.

8. [15 points] The wave function of a particle in a box in the second energy level is given by:

$$\psi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right). \tag{1}$$

Use Schrödinger's equation to determine the energy of the particle in this state. Comapre with our earlier derivation of the energies of a particle in a box.

9. [20 points] Consider again the particle in a box. The wave function for the first energy level is $\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$. The wave function for the second energy level is $\psi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$. For this problem, feel free to use the Wolfram online Integrator.
   (a) If the wave function is in the first energy level, find the expectation value for the position, $\langle x \rangle$. Discuss.
   (b) If the wave function is in the second energy level, find the expectation value for the position, $\langle x \rangle$. Discuss.
   (c) Suppose the wave function is in a superposition of states, $\Phi = A(\psi_1 + \psi_2)$. Normalize Φ and find A.
   (d) Using your result from the last part, find the expectation value for the position of the particle in the superposition state. Discuss your results.

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	10	10	20	10	10	15	15	20	120
Score:										

Last revised on February 22, 2010.